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Exploring Errors and Misconceptions in Differentiation: A Case Study of Advanced Level Students in Zimbabwe

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Abstract

Students' errors help teachers to tease out misconceptions and decide on what intervention strategies to adopt and use to challenge these alternative conceptions. The purpose of the study was to explore A-Level students' errors and misconceptions when solving problems in differentiation. A largely qualitative case study strategy was adopted. The case was made up of Two A-Level teachers and 25 students. After covering the topic differentiation, a test was administered. Qualitative data was collected using the test scripts and syllabus documents, questionnaire and focus group discussions. Content analysis was used to reveal errors and misconceptions when solving problems in differentiation. The common errors and misconceptions displayed by A-Level students were largely procedural when they failed to use the quotient rule, chain rule and power rule in finding derivatives. Our study found out that errors and misconceptions when using the power rule were adding a one to the power part instead of subtracting, bringing down the power and mistakenly adding a one to the power, failure to integrate functions, failure to separate variables correctly, failure to find the required derivative and using wrong laws to solve problems in differentiation. These errors and misconceptions were, possibly originating from students' lack of prior knowledge of differentiation, laws of logarithms, proportionality and integration and rate of change. We recommend further research using larger population and samples.

Keywords: errors, misconceptions, differentiation, derivatives

1. Introduction

Errors and misconceptions in mathematics are important to both teachers and students. Revealing students' errors help teachers to tease out misconceptions and decide on what intervention strategies to adopt and use to challenge these alternative conceptions. Students, through reflection are given opportunity to explain their thoughts, revise and develop deep understanding of concepts. Analysis of errors, together with correctly worked solutions, is a useful tool for increasing mathematical understanding (Rushton, 2018).

2. Literature Review

Students make errors and display misconceptions when learning derivatives (Siyepu, 2013; Zimbabwe Schools Examinations Council, 2010, 2013, 2016, 2017). Numerous studies of misconceptions have been reported, covering different and related topics in Mathematics e.g., linear algebra (Kazunga & Bansilal, 2018), quadratic equations (Makgakga, 2016; Mutambara, Tendere, & Chagwiza, 2020; Kshetree, 2020); vector subset space (Mutambara & Bansilal, 2021), misconceptions and remedies (Ojose, 2015), differential equations (Habre, 2000; Makonye, 2010, 2014), differentiation (Orton, 1983); derivatives (Zandieh, 2000; Maharaj, 2013; Siyepu, 2013, 2015). Errors and misconceptions are related but different. An error is a mistake, slip and deviation from accuracy (Makonye, 2014) and can be systematic or unsystematic (Ricomini, 2005; Luneta & Makonye, 2010). On one hand, unsystematic errors are defined as intended, non-recurring wrong answers which learners can readily correct by themselves. On the other hand, systematic errors are recurrent wrong responses methodically constructed and produced across space and time. Errors made by students can sometimes provide insights into students' understandings about a particular concept or skill (Luneta & Makonye, 2010).

Misconceptions are conceptual structures, which make sense in relation to one's current knowledge but not aligned with conventional mathematical knowledge (Siyepu, 2013; Luneta & Makonye, 2010). A misconception is a misapplication of a rule, an over or under generalization or alternative conception of the situation (Hansen, 2006). When we look at misconceptions as alternative explanations, we create room for students to take us through their thinking and revise their understanding of ideas.

Various ways of grouping errors have been reported in literature. Radatz (1979) reported 5 classes of errors: first, semantics of mathematics or language difficulties; second, difficulties in iconic and visual representation of mathematical knowledge; third, deficiency in requisite skills and knowledge; fourth, incorrect associations; and fifth, application of irrelevant rules. Orton (1983) reduced the classes to three categories: structural error; arbitrary error; and executive error. By structural error is meant failure to appreciate the relationships involved in the problem. Orton (1983) described arbitrary error as failure to take into account the constraints laid down in what was given. By executive error, Orton (1983) meant failure to carry out manipulations. Similarly, Cark (2012) reports three groups of errors; operator, applicability and executive. The three common types of errors are conceptual, procedural and technical (Kiat, 2005; Othman, Khalib, & Mata, 2018; Tendere & Mutambara, 2020). Siyepu (2015) reports four groups of errors, namely: conceptual, procedural, interpretive and linear extrapolation errors. Students display conceptual errors through failure to grasp concepts and relationships in a problem. A procedural error is failure to carry out manipulations or algorithms (Siyepu, 2015), what Orton (1983) called executive error. Interpretive errors occur when students wrongly interpret a concept due to overgeneralisation of the existing schema (Siyepu, 2015) and this is closely related to arbitrary error (Orton, 1983). Linear extrapolation errors occur when students overgeneralised the property which applies only when f is a linear function (Siyepu, 2015). Errors may be due to procedural or instrumental understanding, which is like knowing rules without reason (Skemp, 1976; Hiebert & Lefevre, 1986). Lack of relational understanding, which attends to both knowing what to do and why (Skemp, 1976), is another cause of errors. Whether we should teach rules first followed by reasons or vice versa is like asking what comes first 'chicken or an egg' (Long, 2005). These two groups of errors, conceptual and procedural, were also used by Makgakga (2016).

Misconceptions are often embedded in errors displayed. Literature consulted show that there are overlaps and similarities in the different ways of grouping errors (Radatz, 1979; Orton, 1983; Siyepu, 2015). Furthermore, distinction between an error and a misconception may be blurred. For example, misapplication of a rule can be viewed as a misconception (Hansen, 2006), yet others classify it as applicability error (Clark, 2012). Carelessness or silly mistakes are examples of technical errors (Godden, Mbekwa, & Julie, 2013; Othman et al., 2018; Tendere & Mutambara, 2020). Misconceptions may be revealed in errors evident in problem solving (Mutambara & Bansilal, 2021; Siyepu, 2013). The current study focused on differentiation.

Students struggle when solving problems in differentiation (Othman, Khalib, & Mata, 2018). Research studies show that students have difficulties in acquiring meaningful understanding of the different constructs of a differential equation (Rasmussen, 2001; Habre, 2000; Rasmussen & Blumefeld, 2007). Solving differential equations encompass both differentiating and integrating. Students find the fusion of differentiation and integration to be 'troubling'. When confronted with a problem to solve a first order differential equation, many students simply proceed to integrate even when they have failed to separate the variables algebraically and formulating a statement involving rates of change.

Researches have established that most students find it difficult to understand mathematics concepts, for example, the derivatives (Rasmussen, 2003; Rowland & Jovanoski, 2004; Zandieh, 2000). In Canada, students had difficulties with important concepts of calculus such as the derivative, variables and functions which also hinder their ability to model and solve rate related problems (Tziritas, 2011; Habre, 2000; Schoenfield & Arcavi, 1988). The challenges in application of a derivative and the difficulties in solving derivative problems have been a stumbling block for students in pure mathematics (Schoenfeld & Arcavi, 1988). In Singapore, students experience difficulties with solving differential problems and its application thereof (Maharaj, 2013). In Zimbabwe, most of the students had challenges in pre-calculus concept such as differential basics which have a strong link with the high school mathematics (ZIMSEC, 2014, 2016, 2017).

Otham et al. (2018) argue that students fail to make full use of their previous learning, yet a strong base in prerequisite knowledge is essential in further studies (Tendere & Mutambara, 2020). Students' difficulties in calculus can be traced back to the fundamental concepts of differentiation and integration. The argument here is that each advanced concept is based on some elementary concepts (Siyepu, 2015). Considering the following rules: multiple constant rule, the sum and the difference rules, the power rule, the chain rule and the quotient rule we argue that understanding the quotient and chain rules may require basic understanding of other rules (Siyepu, 2015). Otherwise, students fall prey to 'rush to the rule' (Barnes, 1995) pitfall. Students who do not understand the reasons for and justification of a procedure face difficulties (Orton, 1983; Siyepu, 2013). Many students may provide a general statement of the chain rule and write down the formula, yet only a few of them can explain the connection between the statement and the mentioned rules (Siyepu, 2015). Thus, study of errors and misconceptions can help to attend to these difficulties.

One possible cause of errors reported in literature is lack of knowledge and understanding (Hudson & Miller, 2006) such that students do not follow correct steps (procedural error), fail to recall a fact required to solve problem (factual error), and not understanding a specific concept (conceptual error). Another possible cause of errors is poor attention and carelessness (Hudson & Miller, 2006). The focus of the present study is to gain a better understanding on how Advanced Level mathematics students understand the concept of differentiation. Research into students' difficulties can be useful because students' conceptual difficulties reveal themselves in errors. Furthermore, it has been found that in many cases, student errors are not simply the result of ignorance or due to carelessness, but are in fact systematic consequence of common weaknesses. There seems to be problems that students face when learning mathematics, particularly differentiation. Therefore, the purpose of this study was to explore A-Level students' understanding of differentiation.

The main research question was 'How do students' previous knowledge affect their learning and understanding of differentiation?' Two sub-research questions used were

- What are the errors and misconceptions displayed by Advanced Level students in solving problems in differentiation?
- What is the pre-requisite knowledge required by students to successfully solve problems in differentiation?

3. Methodology

In the methodology section we explore research design, population and sample, data collection methods, data analysis, trustworthiness of data and ethical considerations.

3.1 Research Design

A qualitative case study was used to explore the problems faced by students when leaning differentiation at Advanced level. Qualitative research is a form of social inquiry that focuses on the way people make sense of their experiences and in this case the researchers wanted to explore the experience of form six students in understanding differentiation. Qualitative case study according to Baxter and Jack (2008) is a research strategy that helps in the exploration of a phenomenon within some particular context through various data sources and it undertakes the exploration through a variety of lenses in order to reveal multiple facets of the phenomenon.

3.2 Population and Sample

The targeted population of this study consisted of 25 form six Mathematics students, at a high school in a district located in the northern part of Zimbabwe and their respective teachers. The form 6 class consisted of 15 boys and 10 girls who were doing pure mathematics at Advanced level and had also covered the topic of differentiation under calculus. Convenient sampling was used to select 25 A-Level students at one school. Two teachers, who taught Advanced Level Mathematics, were purposively sampled. Dealing with a small group helped the researchers to get feedback on time and also to adhere to COVID-19 restrictions.

3.3 Data Collection Methods

Data were collected using document analysis, focus group discussions, questionnaires, and interviews. First, using test scripts the study used document analysis to find out errors and misconceptions. Second, the respondents who were A-level students, were asked to express their opinions, feelings and attitudes in separate focused group discussions. Furthermore, teachers were asked to complete a questionnaire seeking their views about students' understanding of the concepts of differentiation. A semi-structured interview was used to follow up on issues emerging from document analysis and teacher questionnaire. The data from teachers, concerning students' misunderstanding was used to establish whether students and teachers shared the same views or that they had differing opinions.

3.4 Data Analysis

Document analysis involved reading written scripts, coding and labelling errors and misconceptions, and creating themes. Interviews were audio recorded, transcribed, labelled and coded emerging issues in order to explore challenges faced by students in differentiation questions. Other researchers have used similar methods (test scripts, interviews, and focus group discussions) e.g., Siyepu (2013), Otham et al. (2018), and Tendere and Mutambara (2020). Error analysis process involves collecting a student's work, asking the student to talk through (or think aloud) the work, recording responses, looking for patterns, describing patterns, and interviewing student to confirm responses (Lai, 2012).

3.5 Trustworthiness of Data

Several methods were used to collect data achieving triangulation. As a way of ensuring trustworthiness of data researchers used member checking in focus group discussions.

3.6 Ethical Considerations

In this research, informed consent was observed through explaining to respondents the purpose of the study and assuring confidentiality and anonymity. Thus, the respondents participated in the interviews, questionnaires and tests fully aware of the purpose of the study and also school authorities granted the researcher permission to interact with them. The respondents were asked to take part on a voluntary basis and they were free to withdraw anytime. The respondents did not use their names and no names are mentioned in this study. Some of the respondents were minors and even though the school acted in loco-parentis, the students were given indemnity forms so that their guardians allowed them to participate in the research.

4. Results and Findings

In this section we present findings and provide answers to the research questions. The study explored errors and misconceptions in differentiation displayed by Advanced level students. We grouped findings under the themes: errors and misconceptions in multiple choice questions, errors and misconceptions when finding the derivative of a power function, errors and misconceptions when using the product and chain rule, errors and misconceptions when using the quotient rule, and pre-requisite knowledge.

4.1 Errors and Misconceptions Evident in Multiple Choice Questions

Figure 1 below is an extract from the multiple-choice scripts. The researchers found out those students who were doing physics recognised dv/dt as representing acceleration. They realized that they were looking for an equation for acceleration rather than velocity.

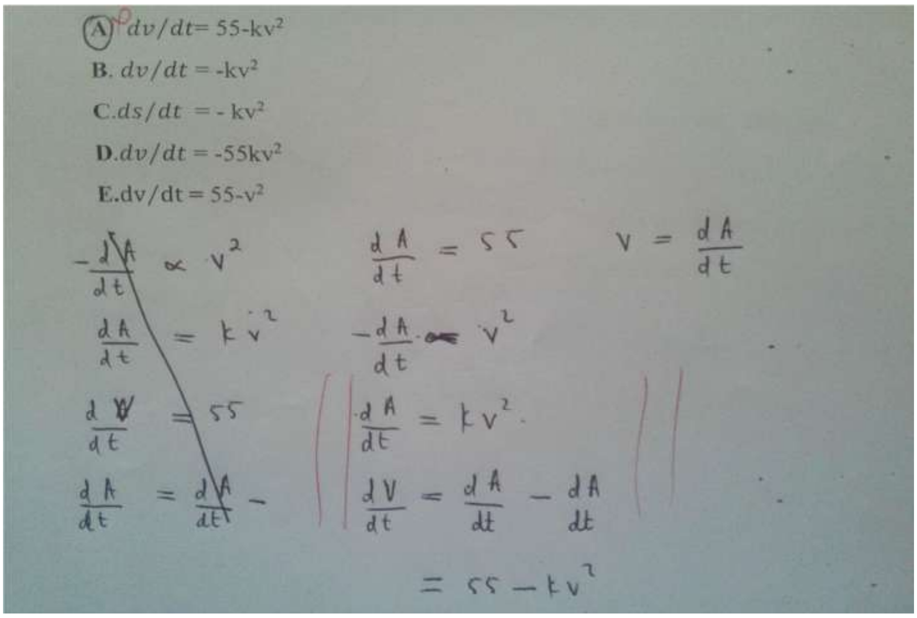

Figure 1. Errors with multiple choice questions

Most of the students performed well on recall questions in the multiple-choice (MCQ) questions. The student who had problems with the derivatives revealed, in interviews, that he always had problems on differentials.

Whilst converting the context into mathematical form, in Figure 1 above, the student failed to interpret word problems particularly amount for rates of changes. It was evident that the student needed to be reminded that the rate of change of velocity is acceleration. Radatz (1979) used the term 'semantics of mathematics' to describe errors to do with language. Similarly, data from interviews was consistent with students' answers to the multiple-choice question 2. Students argued that the differentiation situation was $dvdt$ = 55-kv^2. The car started at 55 km/h and the –kv^2 showed how it slowed down. Various students argued:

'. . . car traveling at 55 km/h - rate at which slowing';

'. . . initial velocity = 55 km/h - something because you will be retarding';

'. . . 55 is initial velocity and velocity decreases from something'

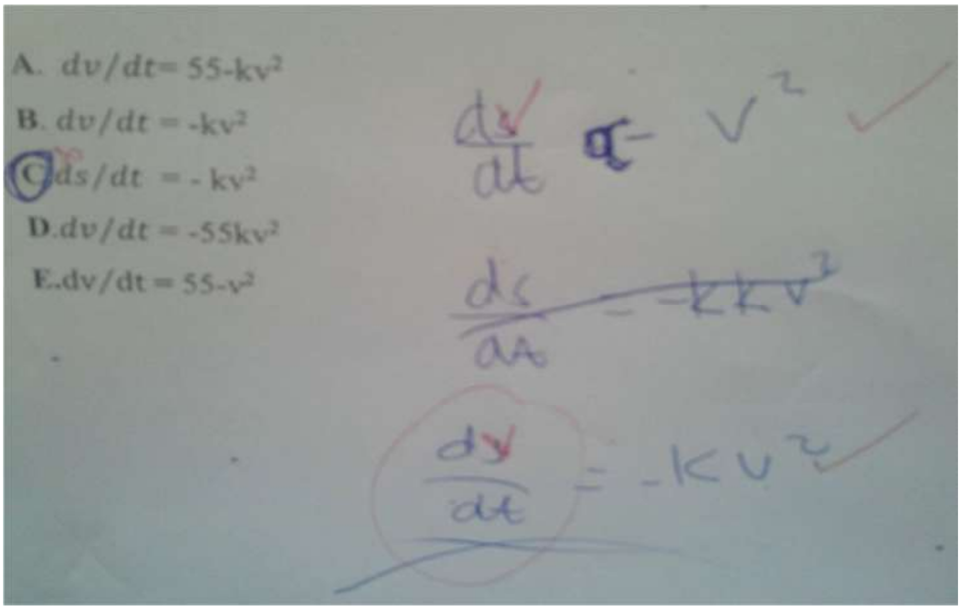

Figure 2. Difficulties on multiple choice questions

During the discussion on the extract of the answer in Figure 2 above, the students revealed that they sought velocity rather than acceleration. We concluded that one misconception by students could be failure to understand the relationship between velocity and acceleration, that velocity changes, and when it changes, that they could determine rate of change (acceleration) considering differences between final velocity (v) and initial velocity (u) over time taken for the change to occur. Our findings are similar to what Radatz (1979) described as incorrect associations, Orton (1983) used the term 'structural errors' and Siyepu (2015) classified such errors as interpretive errors. Difficulties with concepts of calculus hinder students' ability to model and solve rate related problems (Tziritas, 2011; Habre, 2000; Schoenfield & Arcavi, 1988).

4.2 Errors and Misconceptions When Finding the Derivative of a Power Function

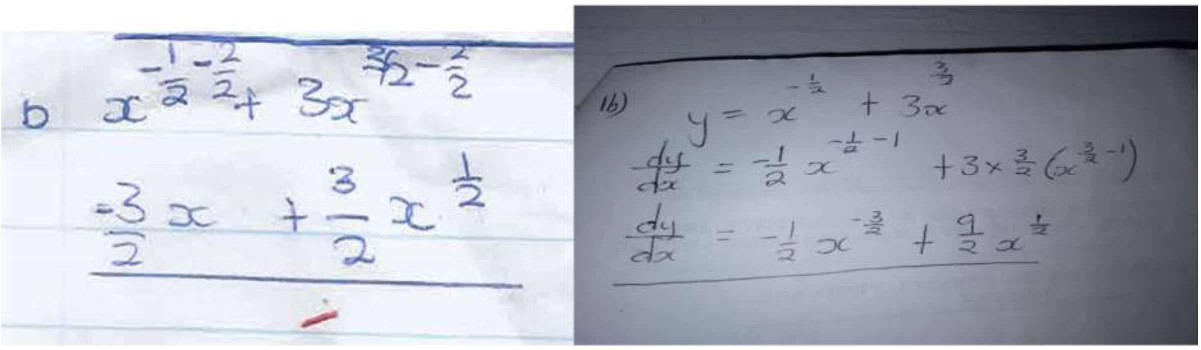

Figure 3. Error on finding the derivative of a power function on the left side and the correct working on the right side

In Figure 3 above the error is that on $x^{-1/2}$ the student correctly brought down -1/2 but failed to simplify the difference between negative half and 1, that is, (-1/2 - 1) and could not to get – (negative) 3/2. With $3x^{3/2}$, in Figure 3 above, the student correctly subtracted the indices as is done in differentiation. However, the student did not multiply a constant with the power during differentiation procedures. As indicated in Figure 3, instead of multiplying the coefficient 3 with 3/2 to get 9/2, the student got 3/2. During the focus group discussions, the students said that they failed to differentiate because they had forgotten to multiply the power with the constant. Interviews revealed that omission of essential steps resulted in failure to get correct answers. Other students also pointed out that they did not understand how to differentiate the power function especially those involving fractions at the time when they were taught differentiation. Our findings would seem to suggest that the students displayed deficiency in requisite skills and knowledge and applied irrelevant rules (Radatz, 1979). The errors are similar to arbitrary (Orton, 1983) or procedural errors (Siyepu, 2015).

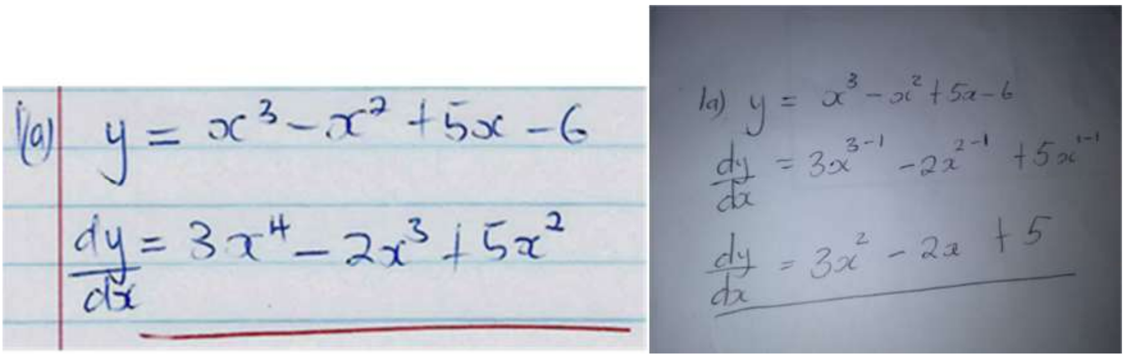

Figure 4. Adding 1 to the index instead of subtracting, and example of procedural errors on differentiation

In Figure 4 above, the student made an error of adding 1 to the index instead of subtracting. This is an example of a procedural error (Siyepu, 2015).

4.3 Errors and Misconceptions When Using the Product and the Chain Rule

In Figure 5 below the errors on product rule originate from failure to remove brackets (x – 3)(2x + 7). The student got $2x^2 - x - 21$ instead of $2x^2 + x - 21$. Such errors have been classified as procedural (Siyepu, 2015) or executive errors (Orton, 1983).

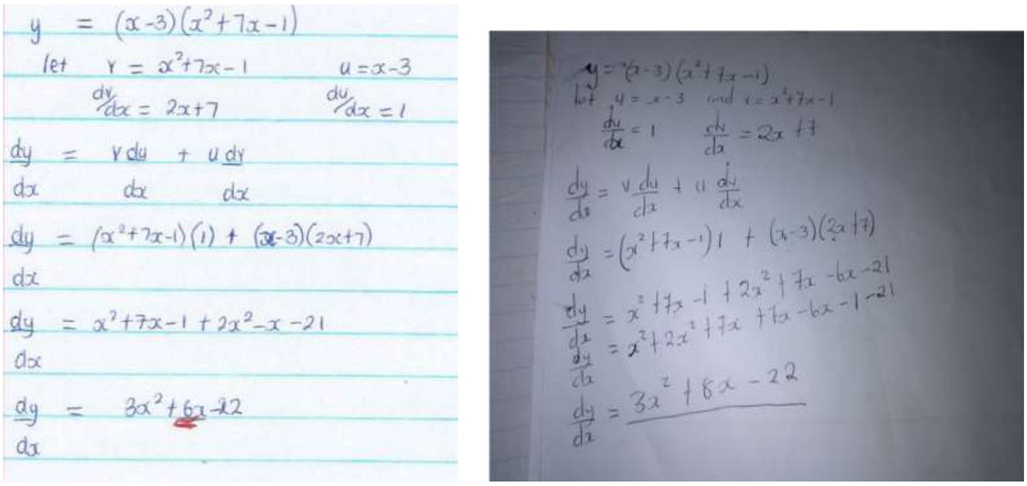

Figure 5. Errors on product rule on the left side and the correct solution on the right side

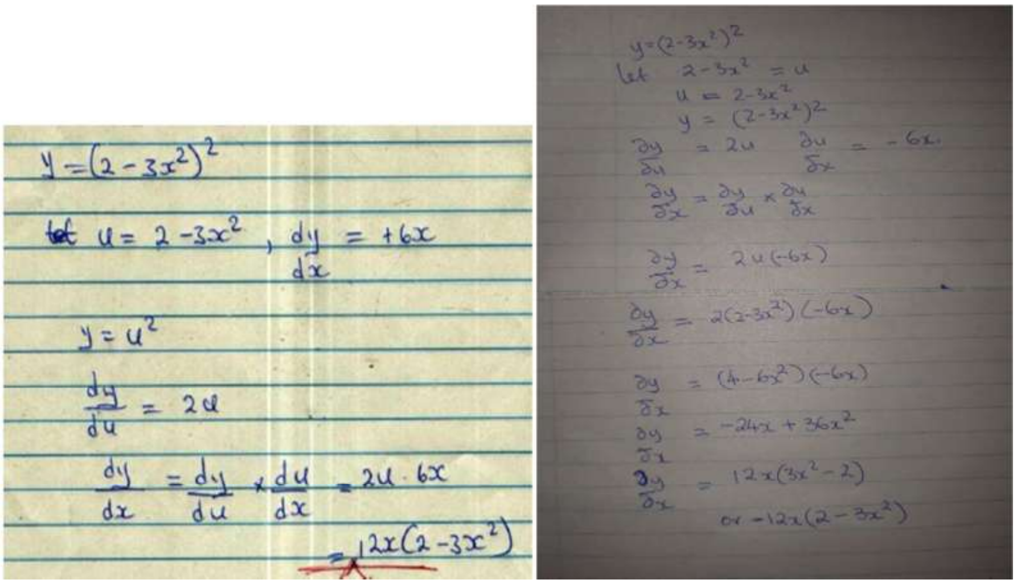

Figure 6. Errors on chain rule on the left side and the correct solution on the right side

In Figure 6 the errors on chain rule are evident. The student made an error on dy/dx = +6x whereas the correct step is du/dx = -6x, which would give a correct solution of dy/dx = $-12x(2 - 3x^2)$ or $12x(3x^2 - 2)$. We deduced this to be a structural error (Orton, 1983).

From the questionnaires, Teacher A highlighted that, students fail to recall concepts they have done in previous studies that are related to differentiation. Teacher B explained that students have misconceptions and procedural errors especially when carrying out chain rule and quotient rule. Teachers agreed that students had faced challenges in substituting the function and believed that most students think that on $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ the du cancels each other to give $\frac{dy}{dx}$. Yet, the cross-cancel principle does not apply (Siyepu, 2015). Students used wrong laws to solve the problem. This is what Radatz (1979) described as application of irrelevant rules. The same error could be viewed as executive (Orton, 1983) or procedural error (Siyepu, 2015). Correct interpretation of chain rule, requires students to find derivative with respect to u first and then find derivative with respect to x (Siyepu, 2015).

Teachers believed that differentiating trigonometrical functions requires a strong background of trigonometrical identities. They believed that students who could not recall what they learnt on trigonometrical identities failed to find the required derivative. Failure to use previous learning is not something new e.g., Otham et al. (2018) and Tendere and Mutambara (2020). Teacher A used tests to identify if students had understood the topic of differentiation and based on the results revisited the concepts students had misunderstood.

We noted that most students failed to understand why the constant is put on the right-hand side. During the follow-up interview one student argued that: "There is no proper reason, why we put the constant A on the right-hand side". Similar to what Siyepu (2015) classified as conceptual errors and Orton (1983) used the term structural error.

Some students had difficulties dealing with the constant of integration, leaving it as part of the exponential. Our research findings confirm results from Camacho-Machin et al. (2012), Rasmussen (2010), Anslan (2010), and Habre (2000) that most students when using algebraic method fail to integrate functions and separate variables correctly.

4.4 Errors and Misconceptions When Using the Quotient Rule

The students who presented incorrect answers were asked to describe the problems they faced when using quotient rule. They had difficulties in choosing functions to represent **v** and also to represent **u**. Further, some students found computing derivative of **u** and **v** to be a challenge. They failed to realise that factorising and simplifying algebraic terms was also very important when finding a derivative. Others said that the procedures and process of choosing **u** and **v** was confusing. They did not know whether to differentiate one of the two or integrate. They found the integrating factor to be complex especially carrying out derivative of **v** and **u**.

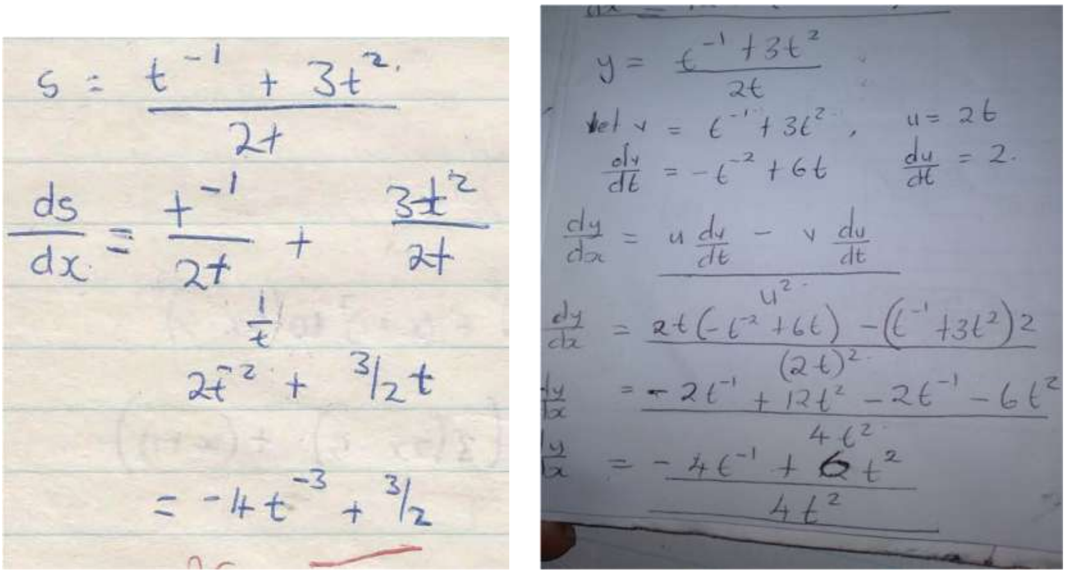

Figure 7. Errors on quotient rule on the left side and correct solution on the right side

As shown in Figure 7, students displayed errors and misconceptions when using quotient rule. In the interview, the students explained that they tried to avoid differentiating a function using quotient rule by trying to apply the laws of indices. Yet, this was a wrong procedure to follow (Siyepu, 2015). Students admitted that they had not mastered the concept of using the quotient rule in differentiation. Rather they found it easy to find derivatives. Those students who got wrong answers found procedures hard to master especially carrying out various derivative of **v** and **u** on the numerator and a $\boldsymbol{v^2}$ on the denominator. Students were confused to such an extent that they failed to answer the question.

Data revealed that students failed to acknowledge whether to add or subtract the numerator with $v\frac{du}{dx}$ or $u\frac{dv}{dx}$ and did not know which of those components came first on the numerator. We deduced that such errors were similar to incorrect associations (Radatz, 1979), arbitrary errors (Orton, 1983), and interpretive errors (Siyepu, 2015). They lacked understanding possibly a consequence of learning through memorising. Our findings resonate with literature that those students who rely on rote learning lack the understanding of the why and how the rules and formulas are derived (Barnes, 1995; Orton, 1983; Cohen & Manion, 2007; Siyepu, 2013).

We found out that students displayed errors and misconceptions in using the power rule. They were making errors in multiplying the constant with the power and also in subtracting the index by 1 during differentiation. Their misconception was adding the indices something done when integrating. In the chain and quotient rule, the idea of choosing a **u** and a **v** seemed to be the error and misconception. Students who displayed errors and misconceptions were cancelling out $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ and thought that the **du** cancels each other to find $\frac{dy}{dx}$.

4.5 Errors and Misconceptions When Finding out the Derivative of Exponentials

Most students failed to find the derivative of exponentials, for instance, the students explained that $y = In\,\frac{1}{x}$ requires one to first simplify $\frac{1}{x}$ to x^{-1} , followed by using the law of logarithm which state that $x^{-1} = -1In\,x$, and finally finding the derivative of an exponential function. Students fail to apply various concepts of finding the derivative of an exponential. Students explained that they had forgotten what they learned at O-Level concerning the laws of logarithms and simplifying exponentials functions. Thus, students had difficulties linking logarithms to natural logarithms.

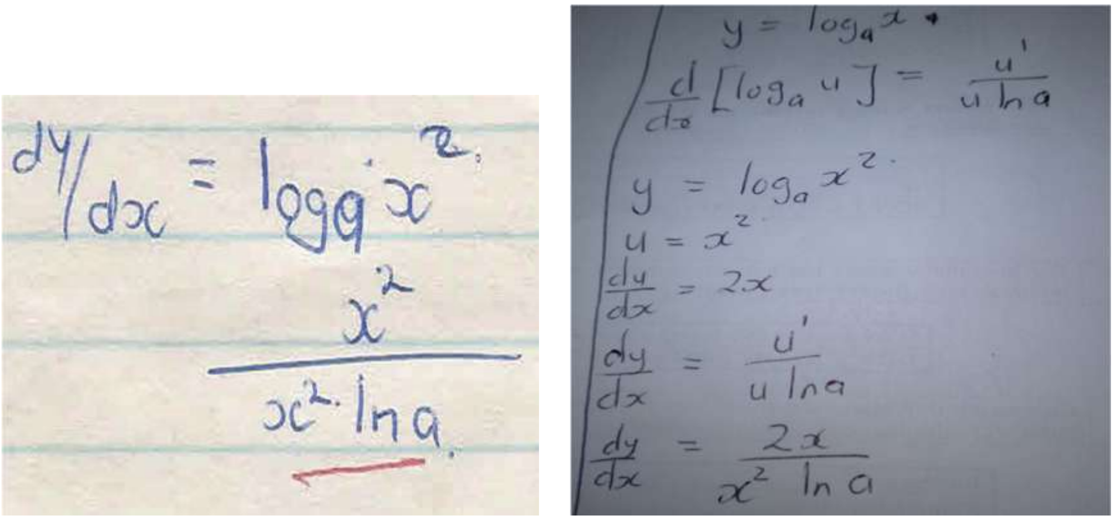

Figure 8. Errors on exponential function on the left side and correct solution on the right side

In Figure 8 above, the students failed to simplify the function before applying the derivative. Interviews revealed that students had difficulties finding derivative of the exponentials. It seems students lacked a firm understanding of exponentials. Some students viewed differentiation as a totally different topic from exponentials. These students did not attempt finding the derivative of an exponential function. Students failed to first simplify the expression before they find the derivative. When students did not understand the situation and failed to differentiate the exponential function, the researchers interpreted this to mean profound misunderstanding of the situation (Luneta & Makonye, 2010), difficulties in iconic and visual representation of mathematical knowledge (Radatz, 1979).

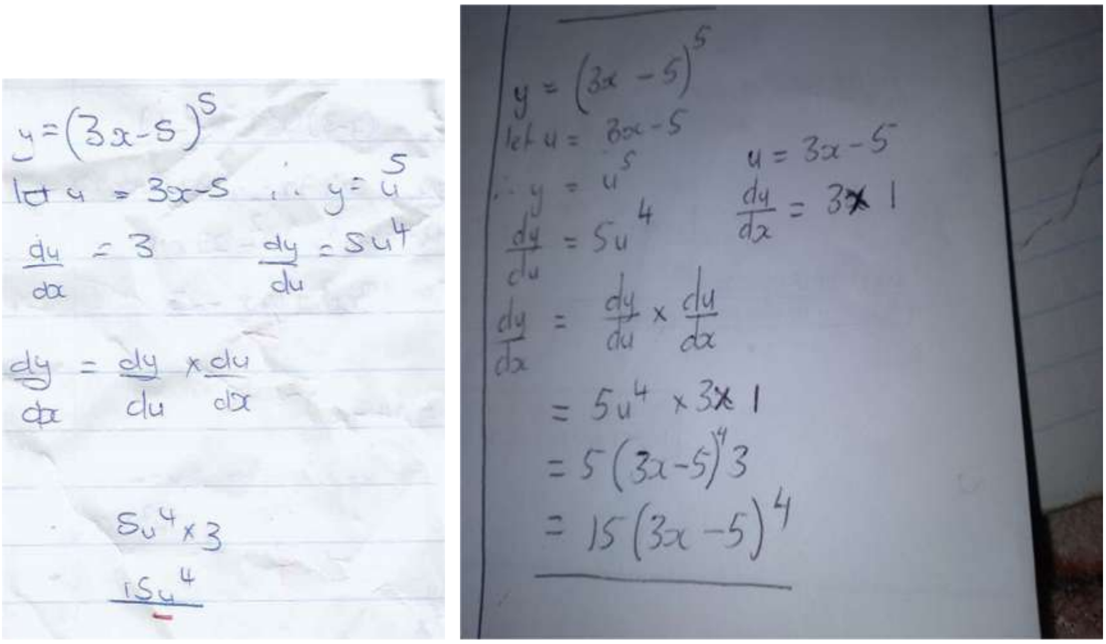

Figure 9. Students' performance on differential equation of the form $(\boldsymbol{ax+b})^{\boldsymbol{n}}$ on the left side and the correct solution on the right side

In Figure 9 above, the student left the differentiated result incomplete when trying to use the chain rule. Using u = 3x-5 stated by the student, all what the student needed was a further step to substitute u with 3x-5. This is an incomplete answer as interpreted. At times students failed because they produced half worked solutions. In the interview students realised, through reflection, that it was important to follow through and complete the procedures to get full marks.

We found out that not understanding exponentials and their derivatives and the interpretation of the terminology in word problems were some of the difficulties that students encountered when solving problems in differentiation. The study found out that the causes of failure to solve problems in differentiation were lack of firm background on dealing with derivatives and simplifying exponential functions, and failure to convert context word problem into mathematical form.

4.6 Pre-requisite Knowledge

Document analysis of the responses on the test scripts and multiple-choice questions on differentiation allowed the researchers to determine pre-requisite knowledge required by students in order to develop the full scope of understanding. The data gathered helped the researcher to identify the students' weaknesses about the topic and infer the pre-requisite knowledge required by students.

5. Conclusion

We used document analysis of the test scripts and interviews to determine errors and misconceptions of derivatives of power, exponential, algebraic and logarithmic functions. A-Level students were able to explain steps in doing chain rule but some failed to justify why they had carried out the particular step in finding the derivative of a function. A-Level students who failed to use the quotient rule, were making errors in choosing **u** and **v** especially when dividing by **v** squared. With respect to the power function, some A-Level students were either not substituting **u** in the final answer or mixing up differentiation and integration in that they would add the power instead of subtracting the power. Others would even subtract the power first then multiply by the coefficient instead of multiplying the power by the coefficient then subtracting the power. Our first conclusion was that the most common errors and misconceptions displayed by A-Level students in solving problems in differentiation are grouped into three; not knowing how to differentiate the power function involving fractions, avoiding differentiating a function using quotient rule by trying to apply the laws of indices, and failing to interpret word problems particularly rates of change.

When A-Level students were using the power rule they encountered difficulties because they had a tendency of adding 1 (one) to the power part instead of subtracting, they were bringing down power and mistakenly adding 1 (one) to the power, they failed to integrate functions, they failed to separate variables correctly, they failed to find the correct derivative and had a tendency to use wrong laws to solve problems in differentiation. When A-Level students were using quotient rule they encountered difficulties because they failed to choose a function to represent **v** and **u,** they got confused on whether to add or subtract the numerator with $v\frac{du}{dx}$ *or* $u\frac{dv}{dx}$ and also which of those components comes first on the numerator. They were wrong in applying the laws of indices when differentiating a function, and multiplying the constant with the power followed by subtracting 1 (one) from the index in differentiation. In the chain and quotient rule, the misconception was cancelling out $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ and this led to $\frac{dy}{dx}$.

Content analysis of the responses to the problems in differentiation allowed us to determine the full scope of students' understanding of the concept. With some A-Level students the weaknesses could be traced back to the basics of calculus thus differentiation and integration and for others it was failure to follow the correct procedures. Some students were able to produce step by step explanation of the problem but were unable to provide justification when asked why they say partialize before differentiating and not differentiate and then partialize. These A-Level students lacked an in depth understanding of the content on partialization.

Some students seemed to be enslaved to the formula booklets. They tried to match a particular form of a function in question with that in the formula booklet. When they failed to match the functions with those in the formula booklet then the end result was failure to attempt the question. This showed that they lacked conceptual understanding. A-Level students failed to solve problems in differentiation because they lacked a firm background on dealing with derivatives and simplifying exponential functions, and lacked conceptual understanding of how to convert context word problem into mathematical form.

Content analysis of test scripts, interview transcripts and questionnaire responses revealed that A-Level students had difficulties linked to variation, algebra, integration, partial fractions and graph sketching of functions. These are the feeder topics of differentiation questions, and therefore, a lack of understanding of these prerequisite topics led to errors and misconceptions found in use of the power rule, chain rule and exponentials. Our second conclusion was that the pre-requisite knowledge required by A-Level students to understand differentiation included variation, algebra, integration, partial fractions and graph sketching of functions, trigonometry, algebra and exponential functions.

6. Recommendations

The study established that it is important to first, devote more time in helping students understand prior knowledge of differentiation. Students needed assistance to understand the basic laws in order to appreciate application of the power rule and the quotient rule. Thus, revisiting the prior knowledge on variation, algebra, integration, partial fractions and graph sketching of functions is essential. For a student to understand differentiation there is need for them to develop an understanding of a function, curve sketching and algebra. A-Level students can then successfully use the previously learned concepts of functions, algebra, partial fractions, and rates of change, proportionality and derivative to solve problems in differentiation. A-Level students may require basic Physics knowledge e.g., acceleration and velocity in modelling of word problems to specific situations.

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Conflict of interest

The authors confirm that there are no conflicts of own interest involved with any parties in the research study.

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