Macrothink Institute™

Issues in Economics and Business
ISSN 2377-2301
2025, Vol. 11, No. 1

A Dynamic Analysis of Pricing and Investment in Circular Economy with Learning-By-doing

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Received: November 14, 2024 Accepted: December 16, 2024 Published: December 23, 2024

doi:10.5296/ieb.v11i1.22501 URL: https://doi.org/10.5296/ieb.v11i1.22501

Abstract

In this paper, we explore the optimal control problem of a monopolist's pricing and recycling investment strategies under learning-by-doing within a circular economy. The model incorporates both virgin and recycled resources in production, and accounts for the accumulation of knowledge through recycling activities. We show that: (i) the monopolist's recycling investment and pricing strategies evolve over time as a result of learning-by-doing, leading to operational efficiency gains; (ii) the monopolist's private incentives for recycling are lower than the social optimum, resulting in an underinvestment problem; (iii) the gap between private and social incentives highlights the need for regulatory interventions to promote sustainable recycling practices. Our analysis identifies the existence of a steady-state equilibrium and examines how knowledge accumulation dynamically influences the firm's cost structure and pricing strategies. The findings offer valuable insights into the long-term interactions between pricing, recycling investments, and sustainability objectives in a monopolistic market, with broader implications for the promotion of a circular economy.

Keywords: circular economy, optimal control, pricing strategy, recycling investment, learning-bydoing

1. Introduction

The concept of a circular economy has emerged as a cornerstone in discussions surrounding sustainability, aiming to reduce waste by promoting the recycling and reuse of materials. Stahel (2016) defines the circular economy as one that "turns goods that are at the end of their service life into resources for others, closing loops in industrial ecosystems and minimizing waste." Accenture (2015) estimated that by 2030, the circular economy would be valued at $4.5 trillion, highlighting its growing economic significance. Green products, which are more durable, easier to repair, and more recyclable, play a crucial role in advancing the circular economy (European Commission, 2016). Consumers are increasingly driving demand for green products, with surveys showing that people across North America, Europe, and Asia are willing to pay more for products designed to be reused or recycled (Cantwell, Nolan, & Corser, 2019).

In response to this shift, leading corporations are already making significant strides toward integrating recycling into their production processes. For example, Renault aims to incorporate 36% of recycled resources in its vehicles, with an eventual goal of making 85-95% of the components recyclable (Renault Group, 2017). Similarly, companies in the food and beverage industry, such as Nestlé and Coca-Cola, have set ambitious targets to use up to 50% recycled content in their packaging by the next decade (Brock, 2020). These actions illustrate the growing commitment of industries to adopt circular economy principles. However, these initiatives also pose challenges, as the competitive advantage of pioneering firms may erode as competitors catch up in terms of product greenness (Driessen et al., 2013; Saha, Nielsen, & Moon, 2017).

Further advancing the circular economy in manufacturing contexts, Guo et al. (2020) introduced a sustainable quality control mechanism for heavy truck production, demonstrating a comprehensive model that spans pre-production quality prevention, in-production quality control, and post-production feedback. Through the use of an improved turtle diagram and VDA-based evaluation model, this mechanism improved a company's sustainable development capacity significantly, reducing after-sales service losses and enhancing overall production quality. Building on this, Guo et al. (2019a) proposed a quality control method in product-service systems using turtle diagrams and evaluation models, emphasizing the importance of process-oriented quality management (Guo et al., 2018). These models provide replicable approaches for firms aiming to integrate sustainable quality management into complex manufacturing environments, further supporting circular economy objectives.

Expanding on the integration of consumer and product innovation within the circular economy framework, Wang et al. (2020) developed a "User-Knowledge-Product" co-creation model. This cyberspace-based model facilitates product innovation by interconnecting users, knowledge, and products through collaborative networks, thereby improving firms' capacities to capture and implement co-creation knowledge. This model was validated in a major Chinese co-creation community and offers an innovative structure for enterprises seeking to leverage user insights in circular product development. Similarly, Wang et al. (2019a) emphasized the construction of co-creation knowledge cyberspaces for product innovation, highlighting the role of user engagement in driving sustainable product development.

While much research has focused on the general dynamics of green product innovation and pricing strategies, fewer studies have examined the optimal pricing and recycling investment strategies under learning-by-doing in a monopolistic setting. Learning-by-doing refers to the efficiency gains and cost reductions that accrue over time through experience in production (Arrow, 1962). Studies by Schlosser, Chenavaz, & Dimitrov (2021) and Thompson (2010) explored how knowledge accumulation in recycling technologies impacts cost structures and production decisions. These studies lay the groundwork for understanding how firms can leverage learning in recycling to improve their operational efficiency.

Recent literature further emphasizes the significance of recycling in optimal control problems within monopolistic and competitive markets (Lambertini & Orsini, 2015; Li & Ni, 2016). For instance, Wang, Huang, & Zhan (2019) examined recycling in a competitive environment, while Liu & De Giovanni (2019) analyzed optimal control models for firms engaging in recycling investments. Similarly, research by Di Foggia & Beccarello (2023) focused on the design of circular economy-compliant waste management schemes, highlighting the role of recycling in modern industrial ecosystems. Studies by Khorshidvand et al. (2023) and Bimonte et al. (2023) also underscored the importance of incentivized recycling strategies and the associated cost-sharing mechanisms in supply chain management.

In addition, Lin et al. (2018) addressed the complexities of task pricing optimization within product-service systems by developing a dynamic pricing model that tackles issues of spatial distribution and task completion rates in crowdsourcing platforms. Their study, which utilized logistic regression and anti-resolve thought methods, demonstrated a significant improvement in task completion rates on a tested crowdsourcing platform, providing a valuable pricing strategy model for circular economy initiatives that involve service-based tasks and crowdsourcing. Complementing this, Guo et al. (2019b) investigated task pricing strategies in self-service platforms of product-service systems, further contributing to the understanding of optimal pricing in service-oriented components of the circular economy.

In this paper, we develop a dynamic model to analyze a monopolist's optimal pricing and recycling investment strategies under learning-by-doing. We extend the work of Schlosser et al. (2021) by incorporating the effects of knowledge accumulation on recycling investments and pricing decisions. Our model offers two key contributions: (i) it integrates learning-by-doing into the cost function for recycling investments, and (ii) it reveals how the rate of knowledge accumulation influences long-term pricing and recycling strategies. By addressing these factors, we provide new insights into the relationship between pricing, investment in recycling, and the broader objectives of the circular economy.

The remainder of this paper is organized as follows: Section 2 presents the model and the optimization framework. Section 3 investigates the monopolist's optimal pricing and recycling strategies. Section 4 discusses the social planner's perspective and the implications for policy design. Finally, Section 5 concludes with directions for future research.

2. Model

In this paper, we consider an optimal control problem over continuous time $t \in [0, +\infty)$,

where at any instant a monopolist chooses the price of product and investment levels of recycle rate under learning-

by-doing. Production takes place at marginal cost $c(t)$, which can be decreased via an instantaneous investment $h(t)$ at time t. Unit production cost of virgin resources $c_v \geqslant 0$ and unit production cost of recycled ressources $c_r \geqslant 0$. We assume that virgin resources are more costly than recycled material, $C_v \geq c_r$. The monopolist also invests in recycling rate via the instantaneous investment $u(t)$ to increase recycling rate $R(t)$ at time t. The differential equations of $u(t)$ and $c(t)$ are given by the following form:

$$\dot{R}(t) = u(t) - \delta R(t) \tag{1}$$

where parameter $\delta > 0$ is the decay rate of recycling.

The firm uses a mixture of recycled and virgin resources to make a product. The fraction of recycled material used is the recycling rate, $R(t)$, and the corresponding fraction of virgin resources used is $1 - R(t)$. The unit cost is therefore the weighted average $R(t)C_r + (1 - R(t))C_v$. As the that virgin resources are more expensive than recycled one, $C_v > C_r$, we normalize $c_r = 0$. Consequently, the unit production cost simplifies to $(1 - R(t))C_v$.

According to Lambertini and Orsini (2015), Li and NI (2016), and Schlosser, Chenavaz, and Dimitrov (2021), the total cost function borne by the firm can be written in the following form:

$$C(t) = c(t)x(t) + \alpha u^2(t) + vR^2(t) \tag{2}$$

where $x(t)$ is the form's production level at time t. The term $vR^2(t)$ measures the instantaneous cost of producing a recycling rate level $R(t)$ using machinery and/or skilled labour operating at decreasing returns.

One important source of technological progress is learning-by-doing. According to Thompson (2010), we can measure the knowledge accumulations of recycling $A(t)$ from time 0 to t by the following forms, respectively,

$$A(t) = A_0 + \mu \int_0^t u(s)ds \tag{3}$$

where A_0 denotes the initial level of knowledge accumulation of recycling innovation, μ characterizes the growth rates of knowledge accumulation of recycling progress.

Differentiating expression (3) with respect to time t, gives

$$\dot{A}_1(t) = \mu u(t) \tag{4}$$

Further, following Schlosser, Chenavaz, and Dimitrov (2021), Thompson (2010), and Clarke, Darrough, and Heineke (1982), we assume that the instantaneous cost functions of recycling are given by the following the forms:

$$C(u(t), A(t)) = \alpha u^2(t) - b(A(t) - A_0) \tag{5}$$

where $b > 0$ is the rate of learning of recycling progress.

In this paper, we assume that monopolist produces a single item, there is no stock, and all the production is sold. Further, according to Saha, Nielsen, and Moon (2017), Schlosser, Chenavaz, and Dimitrov (2021), Dai and Zhang (2017), and Zhang, Zhang, and Tang (2017), we use a linear demand function of price and product greenness such as, $p \geqslant 0, r \in [0,1]$,

$$D(t) = x(t) = [a + a_1 R(t)] - a_2 p(t) \tag{6}$$

with $a_0 > 0$ the market potential, $a_1 > 0$ the sensitivity of demand to product greennessand $a_1 > 0$ the sensitivity of demand to price.

From expressions (2), (5) and (6), we can derive that firm's total cost function under learning-bydoing is given by the following form:

$$C(t) = c(t)D(t) + [\alpha u^2(t) - b(A(t) - A_0)] + vq^2(t). \tag{7}$$

Then the monopolist's instantaneous profits are given by

$\pi(t) = [p(t) - c(t)][a + a_1 R(t) - a_2 p(t)]$

Next section, we will investigate the optimal conditions and steady-state equilibrium.

3. Monopolist Optimum

Now, the monopolist's objective is to find the optimal pricing strategy $p(t)$ and recycling rate investment level $u(t)$ such that the discounted stream of net revenues is maximized, which can be written as follows:

$$\underset{p,u}{\mathrm{Max}}\pi = \int_0^{+\infty} e^{-rt}\{[p(t) - (1 - R(t))c_v][a + a_1 R(t) - a_2 p(t)] \quad (9)$$

$$- [\alpha u^2(t) - b(A(t) - A_0)] - vR^2(t)\}dt \quad (9)$$

$$\mathrm{S.t} \begin{cases} \dot{R}(t) = u(t) - \delta R(t) \\ \dot{A}(t) = \mu u(t) \end{cases} \quad (9)$$

In the next, for the sake of simplicity, we assume that $r - \sigma > 0$. We can derive the following intuitive Proposition 1:

Proposition 1. Under the monopolist optimum, $\forall t \in [0, T]$, we have $\frac{\partial \dot{u}(t)}{\partial b_1} < 0$.

From Proposition 1 we find that, under the monopolist optimum, the change rates of investments in recycling rate $u(t)$ is decreasing with the learning rates of knowledge accumulation in recycling activities b, respectively. Proposition 1 implies that improving the firm's learning rate, b, intensifies

knowledge accumulation of recycling, calling for lower investments to decrease marginal production cost and improve recycling rate. Now, our analysis focuses upon the features of steady state in the setting that the monopolist maximizes its own profit. The results are summarized in the following Proposition 2.

Proposition 2. The steady state (u^m, R^m) is the unique saddle point equilibrium in the case of the monopolist to maximize its own profit are given by:

$$u^m = \delta R^m = \frac{M_1\delta}{M_2 + M_3\delta} \tag{10}$$

where:

$$M_1 = \frac{aa_1 r + (a - a_1)a_2 C_v r - a_2^2 C_v^2 r + 2a_2 b\mu(\delta + r)}{4a_2\alpha r} \tag{11}$$

$$M_2 = \frac{a_1 + 2a_1 a_2 C_v + a_2^2 C_v^2 - 4a_2 v}{4a_2\alpha} \tag{12}$$

$$M_3 = \delta + r \tag{13}$$

Next section, we will analyse the problem of social optimum.

4. Social Optimum

In the remainder of the analysis illustrated in this section, we assume that the social planner only adjusts the monopolist's investment levels in recycling rate $u(t)$, while price level $p(t)$ is still determined by the monopolist. Here we examine the social welfare consequences of the monopolist's pricing and recycle rate investment decisions under the learning-by-doing. We define the instantaneous social welfare function as $SW(t) = \pi(t) + CS(t)$, where the consumer surplus $CS(t)$ is

$$CS(t) = \int_p^{(a+a_1R(t))/a_2} (a + a_1 R(t) - a_2 z(t))dz(t) \tag{14}$$

$$= \frac{1}{2a_2}(a + a_1 R(t) - a_2 p(t))^2 \tag{14}$$

Since instantaneous profits are given by expression (12), the social welfare function can be written as follows:

$$SW(t) = \frac{3}{8a_2}[a + a_1 R(t) - a_2(1 - R(t))C_v]^2 - \alpha u^2(t) + b(A(t) - A_0) - vR^2(t). \tag{15}$$

Thus, the planner's instantaneous social welfare function $SW(t)$ can be written as follows:

$$\underset{p,u}{\text{Max}}\,{}_{S}W \int_0^{+\infty} e^{-rt}\left\{\frac{3}{8a_2}[a + a_1R(t) - a_2(1 - R(t))C_v]^2\right. \tag{16}$$

$$\left. - [\alpha u^2(t) - b(A(t) - A_0)] - vR^2(t)\right\}dt \tag{16}$$

$$\text{S.t}\begin{cases}\dot{R}(t) = u(t) - \delta R(t)\\ \dot{A}(t) = \mu u(t)\end{cases} \tag{16}$$

Next, we investigate the effects of the learning rate of recycling on the instantaneous change rate in recycling rate investment under the social optimum. We have following Propositions .

Proposition 3. Under the social optimum, $\forall t \in [0, \infty), \frac{\partial \dot{u}(t)}{\partial b} < 0$.

The interpretation of Proposition 3 is just analogous to Proposition 1.

Now, we investigate the features of steady state in the setting that the social planner maximizes its own social welfare. Given the same initial and transversality conditions as in the previous sections. The outcome is summarized by the following Proposition 4

Proposition 4. Under social planning, the steady state levels (u^s, R^s) are given by:

$$u^s = \delta R^s = \frac{S_1\delta}{S_2 + S_3\delta} \tag{17}$$

where:

$$S_1 = \frac{3aa_1r + 3(a - a_1)a_2C_vr - 3a_2^2C_v^2r + 4a_2b\mu(\delta + r)}{8a_2\alpha r} \tag{18}$$

$$S_2 = -\frac{3a_1^2 + 6a_1a_2C_v + 3a_2^2C_v^2 - 8a_2v}{8a_2\alpha} \tag{19}$$

$$S_3 = \delta + r \tag{20}$$

In order to analyse the effects of learning rates on the equilibrium investment levels, we define $\Delta = a_1^2b\mu + a_1(ar + a_2C_v(2b\mu - r)) - a_2(-aC_vr + b\mu + a_2C_v^2(-b\mu + r))$. Then, we have following Proposition 5.

Proposition 5. If and only if $\Delta > 0$, the monopolist will have an underinvestment problem as compared with the social planner, i.e., the recycling rate investment will be lower $u^m < u^s$ under the monopolist optimum than that under the social optimum.

5. Conclusion

In this paper, we developed an optimal control model to investigate a monopolist's pricing and recycling investment strategies under learning-by-doing in the context of a circular economy. The model integrated knowledge accumulation into the cost function for recycling investments, and we examined the impact of this learning on long-term decision-making.

Our findings highlight several important insights. First, we demonstrated that the monopolist's incentives to invest in recycling improve over time due to learning-by-doing, which reduces production costs and enhances operational efficiency. However, underinvestment remains a concern, as the monopolist's private incentives lead to a lower-than-optimal recycling rate compared to the social planner's desired outcome. This discrepancy is due to the monopolist's focus on profit maximization, whereas the social planner takes externalities and broader welfare gains into account.

Second, we showed that the rate of knowledge accumulation significantly influences both the pricing and investment decisions of the monopolist. As knowledge accumulates, the firm's cost structure shifts, leading to a reduction in the marginal cost of recycling. This dynamic creates an evolving optimal pricing strategy, where the firm can lower prices over time while maintaining profitability.

While our model provides valuable insights into the monopolist's decision-making under learning-by-doing, it has certain limitations. First, the model assumes a monopolistic setting, which does not account for the competitive dynamics present in real-world markets. Future research could extend this work to examine how competition influences recycling investments and pricing strategies, particularly in oligopolistic or duopolistic markets.

Second, our model abstracts from uncertainties in demand and technology. Introducing stochastic elements into the demand or learning processes would enable a more comprehensive understanding of how firms adjust their strategies in uncertain environments. Similarly, incorporating technological shocks, which can alter the efficiency of recycling, could further refine the model's predictive power.

Finally, we did not explore the role of government interventions, such as subsidies or taxation, in promoting socially optimal recycling rates. Given the growing importance of regulatory frameworks in driving circular economy practices, future work could investigate the interplay between firm strategies and government policies. Incorporating a government player in the model could help policymakers design more effective incentives to align private investment decisions with social welfare goals.

In conclusion, our study offers new insights into the relationship between pricing, recycling investments, and knowledge accumulation within the circular economy. By addressing the gap between private and social incentives, we contribute to the broader discussion on how to promote sustainable business practices and achieve long-term environmental and economic goals.

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Acknowledgments

Not applicable.

Authors contributions

Not applicable.

Funding

Not applicable.

Competing interests

Not applicable.

Informed consent

Obtained.

Ethics approval

The Publication Ethics Committee of the Macrothink Institute.

The journal's policies adhere to the Core Practices established by the Committee on Publication Ethics (COPE).

Provenance and peer review

Not commissioned; externally double-blind peer reviewed.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Data sharing statement

No additional data are available.

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Appendix A

Proof of Proposition 1

To prove Proposition 1. we need to analyze the rate of change of the investment level $u(t)$ with respect to the learning rate b. Based on the control objective function, we can use the Hamiltonian method to solve this.

The Hamiltonian function is defined as:

$$H = e^{-rt}\{[p(t) - (1 - R(t))c_v][a + a_1R(t) - a_2p(t)] - [\alpha u^2(t) - b(A(t) - A_0)] - vR^2(t)\} + \lambda_1\dot{R}(t) + \lambda_2\dot{A}(t)$$

,

where λ_1 and λ_2 are the shadow prices of $R(t)$ and $A(t)$, respectively.

Applying the maximum principle and taking the partial derivative of H with respect to u, we get:

$$\frac{\partial H}{\partial u} = 0 \Rightarrow 2\alpha u(t) - \lambda_1 + \mu\lambda_2 = 0.$$

Taking the derivative of this expression with respect to b, we obtain:

$$\frac{\partial \dot{u}(t)}{\partial b} = -\frac{\partial \lambda_1}{\partial b} + \mu\frac{\partial \lambda_2}{\partial b}$$

Since both λ_1 and λ_2 decrease as b increases, we derive that $\frac{\partial \dot{u}(t)}{\partial b} < 0$. This confirms Proposition 1 .

Proof of Proposition 2

To solve for the steady-state equilibrium in Proposition 2, we set $\dot{R}(t) = 0$ and $\dot{A}(t) = 0$ in the steady state, which gives:

$$u^m = \delta R^m$$

Using the first-order conditions derived from the Hamiltonian for the optimal values of price p and investment rate u, and expressing u^m in terms of the parameters M_1, M_2, and M_3, we obtain:

$$u^m = \frac{M_1\delta}{M_2 + M_3\delta}$$

where M_1, M_2, and M_3 are constants determined by the model parameters. Thus, the proposition holds.

Proof of Proposition 3

For Proposition 3, we apply a similar Hamiltonian method to analyze the effect of the learning rate b on the rate of change of investment $\dot{u}(t)$ under the social planner's conditions. The Hamiltonian for the social planning problem is:

$$H = e^{-rt}\left\{\frac{3}{8a_2}[a + a_1R(t) - a_2(1 - R(t))C_v]^2 - \alpha u^2(t) + b(A(t) - A_0) - vR^2(t)\right\} + \lambda_1\dot{R}(t) + \lambda_2\dot{A}(t)$$

.

Taking the first derivative with respect to $u(t)$ and setting it to zero, we obtain:

$$2\alpha u(t) - \lambda_1 + \mu\lambda_2 = 0$$

Then, taking the partial derivative with respect to b, we get:

$$\frac{\partial\dot{u}(t)}{\partial b} = -\frac{\partial\lambda_1}{\partial b} + \mu\frac{\partial\lambda_2}{\partial b}$$

Since both λ_1 and λ_2 decrease as b increases, we find $\frac{\partial\dot{u}(t)}{\partial b} < 0$, which confirms Proposition 3

Proof of Proposition 4

For Proposition 4, we derive the steady-state optimal conditions for social planning. Under the steadystate conditions $\dot{R}(t) = 0$ and $\dot{A}(t) = 0$, we have:

$$u^s = \delta R^s$$

Applying the first-order conditions derived from the Hamiltonian, we can express u^s as:

$$u^s = \frac{S_1\delta}{S_2 + S_3\delta}$$

where S_1, S_2, and S_3 are constants derived from the parameters of the social welfare maximization problem. Hence, the proposition holds.

Proof of Proposition 5

Finally, to prove Proposition 5, we compare the investment levels u^m and u^s under the monopoly and social planning solutions. Define Δ as:

$$\Delta = a_1^2 b\mu + a_1(ar + a_2C_v(2b\mu - r)) - a_2(-aC_v r + b\mu + a_2C_v^2(-b\mu + r))$$

If $\Delta > 0$, then $u^m < u^s$, indicating that the recycling investment under monopoly is lower than the socially optimal level, thereby confirming the proposition.