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Sleep, Worker's Health, and Social Welfare

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Abstract

The aims of this paper are to construct a theoretical model treating sleep as an investment in health capital and analyze how sleep affects a worker's health capital and social welfare. We consider time constraints because sleep only requires time, and we assumed that consumption also requires time. By doing so, we reveal how workers decide their hours slept. Of particular importance is the result that workers store their health capital by sleeping more than the optimal value at a young age, which leads to increased output. Moreover, we identify a complementary relationship between consumption and labor supply. These results show that policies to reduce hours worked could improve workers' health capital and social welfare.

Keywords: sleep, worker's health, social welfare, health investment, sleep--consumption model

JEL classification: I12; I18; I31; I38

1. Introduction

The main purpose of this paper is to construct a new health investment model that includes sleep and analyze how sleep affects worker health. This study focuses on sleep as a health investment, theoretically verifies how workers decide their hours slept in the long run and suggests policies that can improve their health. Additionally, we examine whether improving worker's health simultaneously ameliorates social welfare.

Sleep is an affordable and important health investment for all organisms. Sleep deprivation among workers has emerged as a serious problem in humans. Hirshkowitz et al. (2015) claimed that adults need seven to nine hours of sleep daily to maintain their health. Elementally, the required sleep duration and level of fatigue vary from person to person. For example, in the United States, approximately 30 percent of workers sleep for less than six hours.[1] For some people, this amount of sleep may be enough. However,

[1] NIH(2006), https://www.nih.gov/news-events/news-releases/nih-offers-new-

assuredly, a problem with lack of sleep exists. In response to this problem, the consumption of supplements and analgesics is increasing to improve the ill health resulting from a lack of sleep (Kaufman et al., 2002).

Sleep has long been known to affect personal health and well-being. Men who sleep more than eight hours have been reported to feel happier than those who sleep fewer than six hours (Barry and Bousfield, 1935). Insufficient sleep results in negative mental consequences, including smoking and drinking (Strine and Chapman, 2005), generalized physical discomfort, and back pain (Haack and Mullington, 2005). In addition, retirees reported improved physical and mental health from sleeping longer (Eibich 2015).

Also increasingly evident is that individual health contributes to cumulative benefits to society (Braveman et al., 2014; Murayama et al., 2012). Improved health results in less morbidity from infectious diseases and greater labor productivity (Bloom, 2003). In other words, an increase in health capital positively affects people and society (Becker 2007).

As these previous studies showed, a lack of sleep may hinder health and, thereby, even reduce labor productivity. The economic costs of sleep disorders are also immeasurable (Streatfeild et al. 2021). The policies that governments implement to address these problems and improve workers' sleep are important not only for workers' health but also for social productivity. To do so, we must clearly determine the amount of sleep that workers should receive.

Another important question is how workers determine their sleep and transition hours. Kuroda (2008) showed that sleep duration decreases with age and leisure increases with age. This finding is not unique to Japan; Ohayon et al. (2004) also showed that sleep quality decreases with age. Moreover, Mander et al. (2017) reported that older adults slept for a shorter duration than younger adults did. The decrease in hours slept with aging is an interesting result. Intuitively, aging seems to lead to physical weakness, which leads to increased sleep duration. As shown by the Centers for Disease Control and Prevention (CDC) (2012), other countries also indicated similar results, and such results do not pertain exclusively to Japan. Identifying the reasons for these similar results will help determine the optimal hours worked for each age group of workers.

To resolve these issues, a new model must be created that treats sleep as a health investment. Grossman (1972) modeled health investment, but the health investment factor introduced in this model is mainly medical consumption, such as medical services,[2] and sleep is not considered.

In addition, in the classic leisure and labor model, consumers earn utility from goods and leisure. The time required to consume goods is not included in the time constraint. However, the consumption of goods usually takes time; therefore, we include time of

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comprehensive-guide-healthy-sleep

[2] Grossman (1972) stated that medical care encompasses all market goods that affect health and not only medical goods. In other words, all market goods can be called medical care. However, sleep is not included in medical care because the market does not treat sleep as a good.

consumption in the time constraint by using a method in the present study similar to Biddle and Hamermesh (1990). In addition, we refer to this behavior of consumption taking time as time-consuming behavior and define it as the entirety of time-consuming and money-spending behavior. Additionally, we include sleep in health investments and extend the Grossman model. By doing so, we can fit sleep into an econometric model as a health investment. Moreover, we separate sleep from leisure and also include sleep-in-time constraints, making it possible to analyze the balance between health investment and consumption within a given time constraint.

The principal results from our model are as follows: (1) workers sleep more at a younger age than during their older years; (2) labor supply and consumption are complementary, and labor supply must also increase at the same time if workers want to increase consumption.

Result (1) theoretically explains the transition in sleep duration with age, as reported by Kuroda (2008) and Ohayon et al. (2004). Workers sleep at a younger age to maintain their future health. Moreover, when a firm's production function includes workers' health capital, real wages are an increasing function of their health capital. This equality implies that workers increase their wages by sleeping more at a younger age to store their health capital, thereby increasing future consumption. This result clarifies the previously unexplained transition in sleep duration with age.

The most important result of this study is result (2). In the case of firms imposing overtime work on workers, policies that reduce hours worked improve not only workers' health but also their utility and firms' profits. Therefore, social welfare is improved through a labor reduction policy. In situations with excess labor, workers must sacrifice more sleep to increase consumption on the basis of the complementary relationship between labor supply and consumption. Furthermore, workers' wages decrease because of lower health capital, and the increment in consumption is less than the increment in hours worked; thus, they cannot consume enough. Policies to reduce the hours worked force workers to sleep more by reducing the hours spent on work and consumption. This result is generated by our model, which includes sleep, consumption, and labor in the time constraint. The improved health of workers from more sleep increases production, workers' wages, and consumption. Labor supply and consumption increase simultaneously; however, if workers' health improves, the increment in hours spent on labor becomes less than or equal to the increment in hours spent on consumption. This leads to workers getting enough sleep to maintain their health and sufficient consumption and increases firms' profits. Therefore, result (2) provides the possibility that policies to reduce hours worked improve social welfare.

However, this study rests on several simplifying assumptions. This study ignores workers' determination of short-term sleep and uncertainty, such as their probability of death.

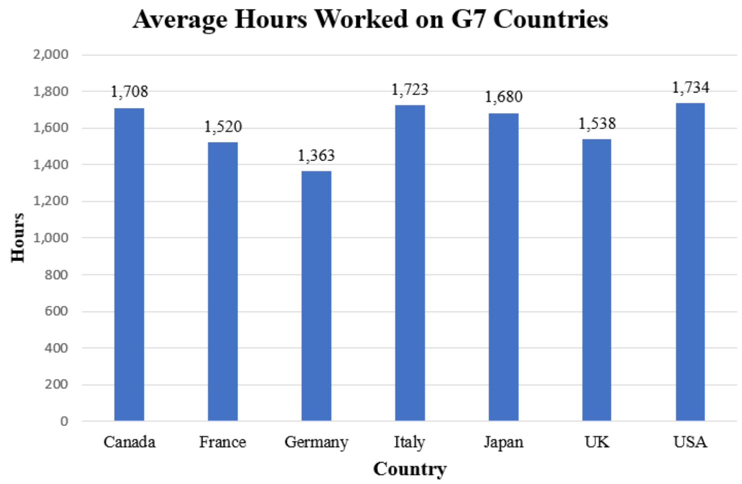

Figure 1. Average hours worked on G7 countries. This figure is created from OECD.Stat.(https://stats.oecd.org/Index.aspx?DataSetCode$=$ANHRS(accessed 2022-06-22))

However, neglecting the essential features of the model presented in this study is erroneous. The most important aspect of this study is that we have provided a constraint on sleep, a factor that is theoretically difficult to deal with, which allows for analysis using the theoretical model. Moreover, considering how to deal with goods that cannot be handled through the market rather than ignoring such goods is important.

The remainder of this paper is organized as follows. Section 2 introduces, in greater detail, empirical research on sleep and theoretical analysis of health investments for creating a new model. Section 3 constructs a model featuring hours slept. In this study, we analyze the budget constraints and hours spent on labor in each of the four cases. We also analyze how policies that reduce hours worked affect workers' health and social welfare. Section 4 presents the results and compares them with those of previous seminal studies. Finally, Section 5 concludes the study.

2. Preliminary

This section presents empirical research on sleep and health investment models to create a new model.

2.1 Empirical Studies

Sleep is an essential factor for a healthy life, and humans sleep when they are tired and want to recover their health. Sleep is a physiological need. If individuals ignore it and stay awake for a long time, doing so reduces not only their health but also their well-being. As previously mentioned, a relationship exists between sleep and happiness. Supporting this relationship, when comparing the hours slept and the happiness levels of people in major developed countries, we see that Japan, which has the lowest sleep duration, has the lowest happiness level in these countries. In addition, countries with longer sleep durations have

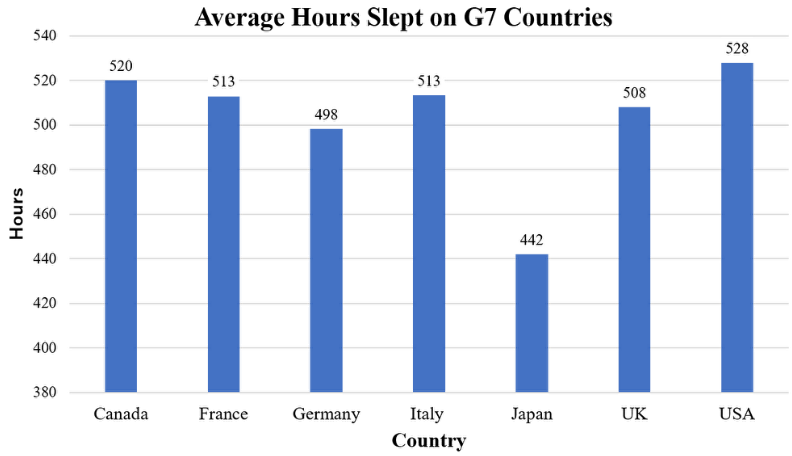

Figure 2. Average hours slept in G7 countries. This figure is created from OECD.Stat.(https://stats.oecd.org/Index.aspx?DataSetCode$=$HRS(accessed 2022-06-23))

more highly ranked happiness levels.[3] Also pointed out is that increasing sleep improves not only health but also well-being (Steptoe et al., 2008).

Quality of sleep is important for health and happiness (Pilcher et al., 1997, Sathyanarayana et al., 2016, Almojali et al., 2017), although people require variable quantities of sleep. Kuroda (2008) examined differences in hours slept according to age and education. A time-use survey by Kuroda (2008) observed hours worked and hours spent on leisure among Japanese workers spanning 1976—2001. Kuroda (2008) showed that sleep duration decreases with age and leisure increases with age. This finding is not unique to Japan; Ohayon et al. (2004) also showed that sleep quality decreases with age. Moreover, Mander et al. (2017) reported that older adults slept for a shorter duration than younger adults did. The decrease in hours slept with aging is an interesting result. Intuitively, aging seems to lead to physical weaknesses, which leads to increased sleep duration. As shown by the CDC (2012), other countries also show similar results that do not pertain exclusively to Japan.

Even though Japanese workers do not work comparably more hours in developed economies (Figure 1), their hours slept are the fewest among these countries (Figure 2). This phenomenon is explained from the phenomenon explained as unpaid overtime work, which is called "service-overtime-working," amounting to 300 to 350 hours per year (Ogata, 2015).[4]

Several studies reported that working overtime can lead to poor health, sleep deprivation, and sleep disorders (Liu et al., 2002; Dembe et al., 2005; Virtanen et al., 2009; Afonso et al., 2017). These results suggest that overtime is one of the most common causes of sleep

[3] Table 2.2 in Sachs et al. (2019)

[4] Kuroda (2008) did not indicate the relation between sleep and health; however, many studies attributed serious health problems to insufficient sleep (Van Dongen et al., 2003; Strine and Chapman, 2005, Grander et al., 2010a). Lack of sleep is also known to negatively affect adolescents (Fredriksen et al., 2004).

deprivation. Additionally, Rosekind et al. (2010) showed that workers' who slept short hours had decreased productivity at a high cost to employers. Section 3.6 considers whether policies that reduce working hours can increase workers' health and social well-being when firms force workers to work overtime.

Sleep plays an important role in human health. In addition, hours slept differ among individuals by age, education, income, and so on. Therefore, clarifying how individuals decide on the number of hours slept is important. However, past empirical studies showed that the differences in hours slept are not fully explained by workers' age and income. Every individual determines his or her sleep duration on the basis of physiological and economic needs. To analyze this, a theoretical model is required.

2.2 Health Investment Model

Grossman (1972) introduced health capital as a way of thinking about health investments. Mushkin (1962), Becker (1964), and Ben-Porath (1967) included health as human capital and showed that investments in human capital increase personal income and labor productivity. In addition, Mincer (1984) and Becker et al. (1990) showed that investments in human capital enhance society's economic growth. Investments in education and job training interact with each other; however, investments in health pertain only to health and are unaffected by other factors. Their investigation showed that education had no direct impact on health. Our investigation showed that education had no direct impact on health.[5]

The Grossman model analyzes health by focusing on health capital but disregards uncertainty; moreover, initial assets do not affect health investments, and the risk of death is not endogenous. Cropper (1977), Wolfe (1985), Dardnoni and Wagstaff (1987, 1990), Lijas (1998), and Jacobson (2000) extended the Grossman model to include uncertainty and changes in mortality and morbidity risks attributable to occupation or education. However, their extensions of the Grossman model consider only medical services as health investments. Sleep is not included in this investment because it is not traded in the market.

Some studies regard sleep as leisure, and classical leisure consumption models categorize leisure as a counterconcept to labor. Such categorizations reveal the conceptual shortcomings of previous models. They cannot categorize sleep as a consumer good because they have no price and are not traded in the market. Therefore, previous studies categorized sleep as leisure because it is neither a consumer good nor labor. Biddle and Hamermesh (1990) created a model that regarded sleep as another time constraint factor that is separate from leisure, whereby individuals can endogenously determine hours slept. However, their study did not consider sleep to be a health investment.

Therefore, we need to create a model that introduces sleep into health investments.

3. The Sleep-Consumption Model

Sleep is affected by physiological needs, but physiological needs alone cannot explain

[5] Acquiring a good education may enhance health through higher income as an indirect effect, but education cannot be said to directly affect good health.

why hours slept are determined by differences between education, age, income, and so on. Therefore, a model that relates sleep to non-physiological factors is needed.

Our model extends the Grossman model by separating sleep from leisure using a method similar to that of Biddle and Hamermesh (1990). The distinguishing feature of our model is that it regards hours slept as an investment in health subject to time constraints. In other words, sleep is not traded through the market; therefore, we treat sleep as an investment in health that entails no money spent but only time. By doing so, we can analyze the effect of sleep on health. Furthermore, we redefine consumption as time-consuming and money-spending behavior---called "time-consuming behavior." Time-consuming behavior should consider all of the time pertaining to consumption; for example, if we eat at a restaurant, we not only consider the meal itself as taking time but also the entire sequence of leaving home, heading to the restaurant, eating, and returning home. Thus, the price of a time-consuming behavior is the cost of the entire behavior. Using this definition of consumption, we can analyze workers' decision making to determine the balance among sleep, consumption, and labor. In addition, depending on the definition of medical care in Grossman (1972), we assume that time-consuming behavior enhances health capital. Therefore, we only consider consumption to be a positive health investment or that the sum effect of the overall consumption behavior on health is positive.

3.1 The Model

Consider a worker who works and lives for T periods. Here, we do not consider workers' retirement period after retirement. We assume that people devote time only to sleep, time-consuming behavior, and labor. Given these conditions, the time not spent working and consumption is used to sleep.

Let S_t, F_t, and H_t denote the three variables of interest: sleep, time-consuming behavior, and health capital, respectively, in period $\mathrm{t} = 1, \dots, \mathrm{T}$. Here, there are n goods; therefore, time-consuming behavior is defined as

$$\mathrm{F_t} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \tau_{\mathrm{i}} \tag{1}$$

where τ_{i} represents the time required for each consumption. Thus, $\mathrm{F_t}$ denotes the sum of the time spent on consumption.[6]

The utility function for workers is:

$$\mathrm{U} = \mathrm{E}\left[\sum_{\mathrm{t}=1}^{\mathrm{T}} \delta^{\mathrm{t}-1}[\mathrm{u}(\mathrm{S_t}, \mathrm{F_t}) + \mathrm{v}(\mathrm{H_t})]\right], \tag{2}$$

where $\delta \in (0,1)$ is a subjective discount rate. Total utility U is defined as the sum of the two utility functions $u(S_t, F_t)$ and $v(H_t)$. Here, $\mathrm{u}(\mathrm{S_t}, \mathrm{F_t})$ is the utility derived from the health investment itself, which is the difference in the utility function between our model

[6] In this model, leisure is included in time-consuming behavior.

and Grossman's model. As discussed in Sections 1 and 2, sleep is associated with personal well-being; therefore, a natural assumption is to introduce it into the utility function. We also explain the reason for splitting the utility function into two parts on the basis of the following assumptions.

Assumptions

(i) $\frac{\partial u}{\partial S} > 0,\ \frac{\partial u}{\partial F} > 0,\ \frac{\partial^2 u}{\partial S^2} < 0,\ \frac{\partial^2 u}{\partial F^2} < 0;$

(ii) There exists S^* such that it satisfies $S < S^*$ if $\frac{\partial u}{\partial S} < \frac{\partial u}{\partial F}$, and $S > S^*$ if $\frac{\partial u}{\partial S} > \frac{\partial u}{\partial F}$;

(iii) $\frac{dv}{dH} > 0.$

Assumption (i) indicates that the utility function is monotonic and concave. Given this condition, workers increase their utility by sleeping or consuming more, and marginal utility decreases with additional time spent on sleep or time-consuming behaviors. Assumption (ii) ensures that the function f is single-peaked with $f(S) = u(S, \overline{T} - S)$. If $\frac{\partial u}{\partial S} < \frac{\partial u}{\partial F}$ (or $\frac{\partial u}{\partial S} > \frac{\partial u}{\partial F}$) holds, workers demand more (or less) time-consuming behavior because they improve (or sacrifice) their utility at that point. By continuing this process, they can attain optimized hours spent on sleep and time-consuming behavior. This single-peak assumption is that an optimal amount of sleep exists for each worker. Assumption (iii) implies that personal utility increases if personal health improves. Assumptions (i) and (iii) explain why the utility function is separated. In terms of health, diminishing utility has no effect. Thus, we divided the utility function into two parts.

Total time, $\overline{T}$, is the sum of hours slept, spent on time-consuming behavior and spent on labor. Then, the time constraint is

$$\overline{T} = S_t + F_t + L_t. \tag{3}$$

Health capital in period t is the sum of the health capital during the previous period and the time spent on sleep and time-consuming behavior minus hours spent on labor:

$$H_t = \phi_{t-1} H_{t-1} + \alpha S_{t-1} + \beta F_{t-1} - \gamma L_{t-1}, \tag{4}$$

where $\alpha > \beta > 0$[7] and $\gamma > 0$. This assumption means that the relationship between health and sleep is stronger than that of health and time-consuming behavior.[8][9] Additionally, ϕ_t denotes the depreciation rate of physical deterioration in period t. Following

[7] As was previously discussed, consumption sometimes restores and sometimes degrades a worker's health. Therefore, parameter β could be positive or negative. For simplicity, we assume the former---that is, $\beta > 0$.

[8] In this model, time-consuming behavior is supposed to be relaxation, eating, travel, and so on.

[9] The appendix considers the case of $\beta > \alpha$.

Grossman's model and, by nature, we assume that health deteriorates at a certain rate per period t. Assuming $\phi_t \leq 1$ for any t is natural because physical health declines with age. Moreover, we assume that initial health capital H_1 is given. $H_t = 0$ signifies death.

The budget constraint is

$$A_t = (1 + r)A_{t-1} + wL_t - pF_t, \tag{5}$$

where $A_t, r, w,$ and p, respectively, are the assets during period t, interest rate, wage rates, and price of time-consuming behavior. Additionally, wL_t represents the income during period t. The term pF_t is a time-consuming behavior expressed in units of price. For simplicity, the interest rate and prices are constant, and we assume that only one consumer good exists. We assume that the initial assets are zero.

Finally, the maximization problem of workers can be formulated as follows:

$$\max_{\{S_t\}_{t=1}^{T}, \{F_t\}_{t=1}^{T}} \quad U = E\left[\sum_{t=1}^{T} \delta^{t-1}[u(S_t, F_t) + v(H_t)]\right]$$

$$\text{subject to} \quad A_t = (1 + r)A_{t-1} + wL_t - pF_t;$$

$$H_{t+1} = \phi_t H_t + \alpha S_t + \beta F_t - \gamma L_t;$$

$$\overline{T} = L_t + F_t + S_t.$$

3.2 Constant Labor Supply with No Budget Constraint

Workers cannot always determine how many hours they work because those hours are set by firms, labor-management agreements, and statutes. Here, we assume that hours worked are set exogenously; therefore, workers can determine the ratio of hours spent on sleep and time-consuming behavior. In this section, we also assume that workers face no budget constraint. They have sufficient wealth or receive sufficiently high wages to ignore budget constraints and can access sufficient goods for consumption.

Because labor supply (hours worked) is an externally determined constant, $L_t = \overline{L}$, we have

$$\overline{T} - \overline{L} \equiv \overline{T}' = F_t + S_t, \tag{6}$$

where $\overline{T}'$ indicates the amount of time available for sleep and time-consuming behavior. Equation (6) allows the workers' utility function to be rewritten as a function of S_t. That is,

$$U = E\left[\sum_{t=1}^{T} \delta^{t-1}[u(S_t, \overline{T}' - S_t) + v(H_t)]\right]. \tag{7}$$

Here, because $u(S_t, F_t)$ is concave, we treat $u(S_t, \overline{T} - S_t)$as a concave function.[10]

By differentiating Equation (7) with respect to S_t, we find the optimal value of sleep at

[10] See Appendix, Lemma 1.

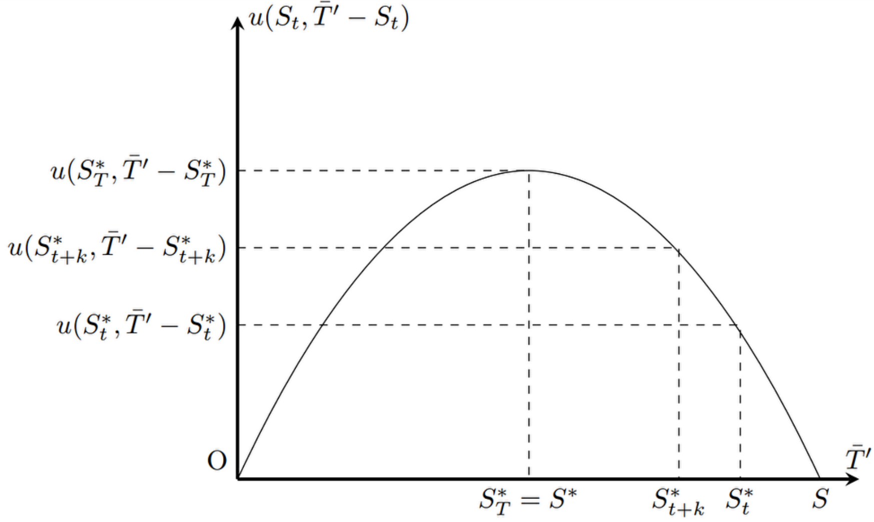

Figure 3 Transition of the optimal hours slept.

period t by

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{du}{dS} + \frac{dv}{dH} \cdot (\alpha - \beta) \left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k \right] = 0. \quad (8)$$

The following relation is used in Equation (8):

$$H_T = \prod_{t=1}^{T-1} \phi_t H_1 + \sum_{t=1}^{T-1} \prod_{k=1}^{T-t+1} \phi_{T-k}(\alpha S_t - \beta F_t - \gamma L_t). \quad (9)$$

Becauseϕ_t, δ^t, and $(\alpha - \beta)$ are positive in Equation (8) and Assumption (ii), the following inequality holds between the optimal value, S^*, and the optimal hours slept for each period, S_t^*, except the final period:

$$S_t^* > S^*.$$

Assumption (ii) reveals the optimal value of the hours slept, S^*. In addition, Equation (8) shows that the optimal number of hours slept during the final period coincides with the optimal value S^*. [11]

$$S_T^* = S^*,$$

thereby resulting in the following inequality:

$$S_t^* > S_T^* = S^*, \quad (10)$$

for all t.

Equation (10) indicates that the optimal number of hours slept during the final period S_T^* is less than that during all other periods. From these results, we obtain Proposition 1.

[11] Therefore, $\frac{\partial u}{\partial S} = \frac{\partial u}{\partial F}$ holds at this point.

Proposition 1. *Without a budget constraint, the optimal number of hours slept, S_1^*, is greater than that in other periods, and $S_t^* > S_{t+1}^*$ holds for each period. Therefore, the following inequalities hold:*

$$\mathrm{S}_1^* > \mathrm{S}_2^* > \cdots > \mathrm{S}_{\mathrm{T}-1}^* > \mathrm{S}_\mathrm{T}^* = \mathrm{S}^*. \tag{11}$$

Proof. See Appendix.

Proposition 1 is expressed in Figure 3, which shows that the optimal hours slept during each period excluding period T is shifted farther to the right side than the optimal value, S^*. Therefore, workers are attempting to maintain their health by sleeping more at a younger age.

3.3 Constant Labor Supply with Budget Constraint

This section introduces the budget constraint in the model in Section 3.2. The budget constraint is

$$A_t = (1+r)A_{t-1} + w\bar{L} - pF_t. \tag{12}$$

Here, income is constant inasmuch as wage rates and labor supply are constant.

Because labor supply is constant, the time constraint is

$$\bar{T}' = S_t + F_t.$$

Using this time constraint, Equation (12) can be rewritten as:

$$\bar{T}' - \frac{(1+r)A_{t-1} - A_t + w\bar{L}}{p} = S_t. \tag{13}$$

Here, the optimal hours for workers to sleep hold when $A_t = 0$. If $A_t > 0$ holds, then workers can improve their utility by increasing their consumption until $A_t = 0$ from the monotonicity of u. In addition, if $A_t < 0$ holds, then workers can increase consumption during this period but must forgo more consumption in the following periods than the increased consumption in the current period because the wage rate is constant and an interest rate exists. Thus, workers choose $A_t = 0$ to maximize their total utility U. Then, Equation (13) can be rewritten as:

$$\bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p} = S_t. \tag{14}$$

To analyze the difference between Section 3.2 and 3.3, we exclude the case in which

$$S_t^* \geq \bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p}. \tag{15}$$

If Equation (15) holds, then the optimal hours slept coincide with the case in Section 3.2.[12] This case is special and holds when initial assets are sufficiently large or the wage rate, w,

[12] See Appendix, Theorem 1.

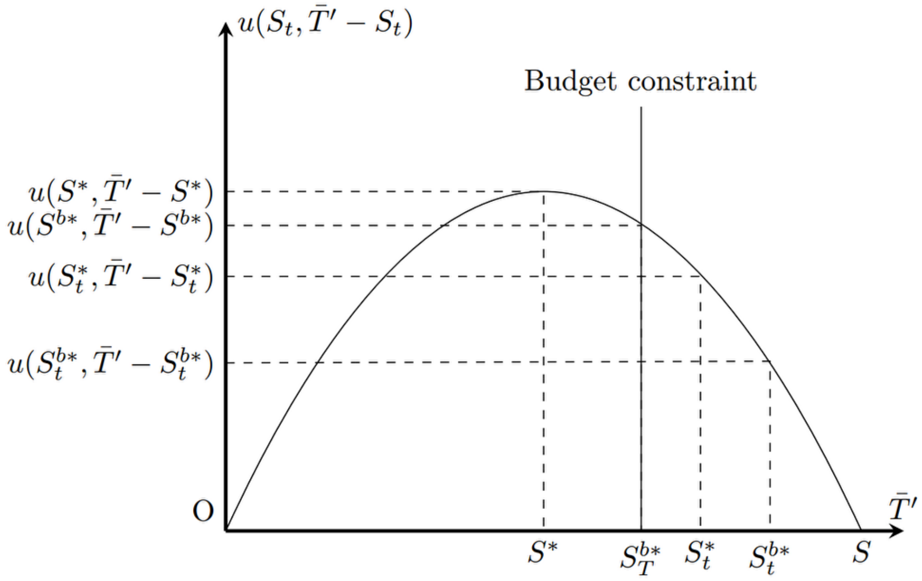

Figure 4. Transition of the optimal hours slept under the budget constraint

is sufficiently large relative to the price, p. On the basis of this discussion, the following inequality holds in the case of a budget constraint:

$$S_t^* < \bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p}. \tag{16}$$

Thus, considering only the following inequality for each period sufficient:

$$S_t^* < S_t', \tag{17}$$

for all periods $t = 1,2, \dots, T$.

This section defines S_t' as the optimal hours slept given a budget constraint, S_t^{b*}, if S_t' satisfies $\bar{T}' + \frac{(1+r)A_{t-1}+w\bar{L}}{p} = S_t'$. We analyze the difference in the optimal value of hours slept given in Section 3.2 and 3.3 by this argument.

From Equations (16) and (17), S_t' exists such that

$$S_t^* < \bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p} \leq S_t'.$$

From the monotonicity of u, the hours slept by workers are consistent, given the budget constraint. Workers' utility improves if the hours slept move slightly left from a point on the right side of the budget constraint, but workers cannot select a point to the left of the budget constraint because hours worked are constant. Thus, we have S_t^{b*} such that

$$S_t^* < \bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p} = S_t^{b*}.$$

From Equation (12) and concavity, the following inequality holds for each period,

$t = 1,2, \dots, T$:

$$A_{t+1} > A_t. \tag{18}$$

Therefore, we obtain Proposition 2.

Proposition 2. *The following inequalities hold in the presence of a budget constraint.*

$$S_1^{b*} > S_2^{b*} > \cdots > S_{T-1}^{b*} > S_T^{b*}.$$

Proof. See Appendix.

In this section, even though labor is constant under the budget constraint, the results of the transition of hours slept are similar to those in Section 3.2. However, in terms of total hours slept, hours slept in the case in which the budget constraint is longer than in the case without. Therefore, when labor is constant and a budget constraint exists, hours slept increases because time cannot be carried over to the next period, and workers cannot fully exhibit time-consuming behavior.

The discussion thus far reveals the following relationship for all periods $t = 1,2, \dots, T$:

$$S_t^* < \overline{T}' - \frac{(1+r)A_{t-1} + w\overline{L}}{p} = S_t^{b*}.$$

Therefore, from Propositions 1 and 2 and the previous discussion, we obtain the following proposition:

Proposition 3. *Given a budget constraint, either of the following inequalities holds for all periods, $t = 1, \dots, T$:*

$$S_t^* \leq S_t^{b*}. \tag{19}$$

Proof. See the Appendix.

Figure 4 illustrates this relation for period t. As shown in Figure 4, the optimal hours slept under a budget constraint exceed those without budget constraints. Therefore, S^* is bounded by the budget constraints (12). In addition, the amount of time-consuming behavior decreases under time constraints. Furthermore, the total utility under a budget constraint is smaller than that without one.[13]

3.4 Flexible Labor Supply with No Budget Constraint

As stated in Section 3.2, workers generally cannot alter the socially determined labor supply at their discretion. However, new modes of working (e.g., freelance and remote

[13] This is trivial. From Propositions 1, 2, and 3, $\sum_{t=1}^{T} S_t^{b*} > \sum_{t=1}^{T} S_t^*$ and $\sum_{t=1}^{T} F_t^{b*} < \sum_{t=1}^{T} F_t^*$ hold. From assumption (ii), at the point at which $\frac{du}{dS} < \frac{du}{dF}$, the utility obtained from time-consuming behavior is greater than that obtained from sleep. Furthermore, the utility obtained from time-consuming behavior near the optimal point $S^* = F^*$ exceeds the utility earned from time-consuming behavior farther from the optimal position. This statement holds true.

work) grant some workers greater discretion to alter the labor that they supply. This section considers that labor supply can be changed, but no budget constraint exists. Although no budget constraint seems unrealistic, analyzing all cases is important.

In this case, workers have no will to work because they have ample money, and working negatively affects their health, as indicated by the definition of accumulated health. Declining work raises their utility, and the quantity of labor supplied during each period must be zero. That is,

$$L_t = 0.$$

Then, workers' utility is

$$U = E\left[\sum_{t=1}^{T} \delta^{t-1}[u(S_t, \bar{T} - S_t) + v(H_t)]\right].$$

Unlike in Section 3.2, workers' accumulated health is represented as

$$H_{t+1} = \phi_t H_t + \alpha S_t + \beta(\bar{T} - S_t). \tag{20}$$

Equation (20) shows that work no longer affects health, and we find the optimal value of sleep in this setting.

Differentiating U with respect to S_t, we obtain the optimal hours slept for each period as:

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{du}{dS} + \frac{dv}{dH} \cdot (\alpha - \beta)\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right] = 0. \tag{21}$$

The value of Equation (21) is identical to that in Section 3.2. Therefore, the discussion generalizes to Section 3.2. If $L_t = \bar{L}$ holds, the workers' total utility, hours slept, and hours spent on time-consuming behavior equal the results in Section 3.2. If $L_t < \bar{L}$ holds, the total utility, hours slept, and hours spent on time-consuming behavior obviously differ. Thus, we obtain Proposition 4.

Proposition 4. *Without budget constraints, workers' utility and health decline if they increase the labor supply*

Proof. See Appendix.

From Proposition 4, we can recast the character of labor as inciting not only disutility but also the detrimental effects on health. We can confirm that working fewer hours improves health and utility but cannot surmise how many more hours workers devote to sleep or time-consuming behavior.

3.5 Flexible Labor Supply with Budget Constraints

This section compares the most realistic case to the outcomes in previous sections. Workers earn income by supplying labor and live through this income. Sometimes, they work overtime with orders from the firm. As mentioned in Section 3.4, more jobs are now available in which workers can freely choose their working time. We analyze such a case.

The maximization problem for workers is:

$$\max_{\{S_t\}_{t=1}^{T}\{F_t\}_{t=1}^{T}} \quad U = E\left[\sum_{t=1}^{T} \delta^{t-1}\left[u(S_t, F_t) + v(H_t)\right]\right]$$

$$\text{subject to} \quad A_t = (1+r)A_{t-1} + wL_t - pF_t$$

$$H_{t+1} = \phi_t H_t + \alpha S_t + \beta F_t - \gamma L_t$$

$$\bar{T} = L_t + F_t + S_t.$$

Substituting the time constraint, $\bar{T} = L_t + F_t + S_t$, into the budget constraint, we have

$$A_t = (1+r)A_{t-1} + w(\bar{T} - F_t - S_t) - pF_t, \tag{22}$$

where assuming the initial asset is as follows:

$$A_0 = 0.$$

Then, the budget constraint is rewritten as

$$A_t = (1+r)A_{t-1} + w(\bar{T} - F_t - S_t) - pF_t \tag{23}$$

$$\Leftrightarrow S_t + \left(1 + \frac{p}{w}\right)F_t + \frac{A_t - (1+r)A_{t-1}}{w} = \bar{T}. \tag{24}$$

From the time and budget constraints, we can rewrite the budget constraint as follows:[14]

$$\bar{T} = S_t + \left(1 + \frac{p}{w}\right)F_t. \tag{25}$$

From Equation (25), by substituting time-consuming behavior into the utility function and health accumulation, we have

$$U = E\left[\sum_{t=1}^{T} \delta^{t-1}\left[u\left(S_t, \frac{\bar{T} - S_t}{\left(1 + \frac{p}{w}\right)}\right) + v(H_t)\right]\right] \tag{26}$$

and

$$H_{t+1} = \phi_t H_t + \left(\alpha - \left(\frac{1}{\left(1 + \frac{p}{w}\right)}\right)\left(\beta - \gamma\left(\frac{p}{w}\right)\right)\right)S_t + \left(\frac{1}{\left(1 + \frac{p}{w}\right)}\right)\left(\beta - \gamma\left(\frac{p}{w}\right)\right)\bar{T}. \tag{27}$$

Thus, we can analyze the maximization problem in this section as the optimization of one variable.

[14] See Appendix, Theorem 3

As in previous sections, differentiating U with respect to S_t produces

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{du}{dS} + \frac{dv}{dH} \cdot \left[\alpha - \left(\frac{1}{\left(1 + \frac{p}{w}\right)}\right)\left(\beta - \gamma\left(\frac{p}{w}\right)\right)\right]\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right] = 0. \quad (28)$$

From Equation (28), we obtain Proposition 5.

Proposition 5. *If the hours worked are flexible under a budget constraint, the following inequalities hold.*

$$S_1^* > S_2^* > \cdots > S_{T-1}^* > S_T^* = S^*.$$

Proof. See Appendix.

In this case, the result of the transition of hours slept remained the same as in the previous sections, with workers sleeping more at a younger age. As in previous sections, Proposition 5 indicates that workers attempt to maintain their health throughout their lives by sleeping more at a younger age.

Regarding optimal time-consuming behavior and labor, the following relations hold by the ratio between the wage rate and price:

$$\begin{aligned} L_t^* &> F_t^* \text{ if } p > w \\ L_t^* &= F_t^* \text{ if } p = w \\ L_t^* &< F_t^* \text{ if } p < w. \end{aligned} \quad (29)$$

From Proposition 5 and Equation (29), Proposition 6 is obtained.

Proposition 6. *If L is flexible under a budget constraint, workers' optimal hours slept are less than or equal to those when L is constant.*

Proof. See the Appendix.

Finally, Proposition 7 is obtained.

Proposition 7. *If L is flexible and a budget constraint exists, then more time-consuming behavior also increases labor supply and vice versa.*

Proof. See the Appendix.

Proposition 7 asserts that because labor and time-consuming behavior are complementary, increasing time-consuming behavior also increases labor and vice versa. Because of this fact and time constraints, reducing labor will decrease consumption, increase sleep, and improve health. This fact is important. If a firm forces its workers to work excessively, then the workers increase their consumption because they earn more income by supplying more labor. However, doing so reduces sleep and increases the damage from labor, which reduces health capital. An increase in consumption increases the utility function of u, but a decrease in health capital decreases the utility function of v. Here, we do not know the impact of increasing labor on U. However, if the labor supply decreases, sleep increases and health improves, which lead to increases in both u and v. If this effect is significant, then U will increase. Therefore, we should consider the effect of a labor reduction policy on workers' health and utility. The effect of policies that reduce working hours on the health and social welfare of workers is discussed in the next section.

3.6 Effectiveness of Labor Reduction Policy

Proposition 7 shows that reducing labor also reduces time-consuming behavior and that workers' health can be improved by increasing sleep. Here, we consider the impact of a policy of decreasing labor on social welfare when firms force workers to work excessively.

In this case, the firm's production function is:

$$F(K_t, H_t L_t) = Y_t, \tag{30}$$

where K_t and Y_t are capital in period t and output in period t. Assume that capital is constant, that is, $K_t = K = 1$. Then, (30) becomes

$$F(H_t L_t) = Y_t. \tag{31}$$

To analyze simply, we specify the production function as follows:

$$Y_t = (H_t L_t)^{(1-\alpha)}. \tag{32}$$

From the definition of time-consuming behavior (1), a relationship exists between tY_t, which is the output Y_t multiplied by the time t to consume it, and the time-consuming behavior F_t, as follows:

$$tY_t = F_t. \tag{33}$$

Equation (33) requires that all production be fully consumed by workers during their consumption period F_t. In other words, as production increases, a corresponding increase in the amount of time spent on time-consuming behavior occurs within the time constraint. Indeed, time-consuming behavior is a function of labor and real wages. This result is also obvious from[15]

$$F_t = \frac{w}{p} L_t. \tag{34}$$

Using Equation (33), the market-clearing condition with $t = 1$ is:

$$Y_t = F_t. \tag{35}$$

The profit maximization problem for firm is:

$$\max_{\{L_t\}_{t=1}^{T}} \qquad \Pi = \sum_{t=1}^{T} \pi_t = \sum_{t=1}^{T} (pY_t - wL_t) \tag{36}$$

[15] See Appendix, Corollary 2.

$$\text{subject to} \qquad Y_t = (H_t L_t)^{(1-\alpha)}.$$

We assume that $\pi'_{L_t} > 0$, $\pi''_{L_t} < 0$. Differentiating Π with respect to L_t, we obtain:

$$\frac{\partial \Pi}{\partial L_t} = (1-\alpha)\frac{H_t^{(1-\alpha)}}{L_t^{\alpha}} p - w = 0$$

$$\Leftrightarrow \frac{w}{p} = (1-\alpha)\left(\frac{H_t^{(1-\alpha)}}{L_t^{\alpha}}\right) \tag{37}$$

$$\Leftrightarrow L_t = \left[\left(\frac{p}{w}\right)(1-\alpha)H_t^{(1-\alpha)}\right]^{\frac{1}{\alpha}}. \tag{38}$$

Equation (38) indicates a labor demand function, $L^D = L^D\left(\frac{w}{p}, H_t\right)$, which is a decreasing function of the real wage and an increasing function of health capital. From Equation (37), real wages decrease as workers' health declines. Moreover, because real wages are a decreasing function of labor, and health capital is also a decreasing function of labor, firms can decrease real wages by imposing excess labor on workers. If the effect of the demand for labor from lower real wages exceeds the effect of improved health capital, then the firm's demand for labor will increase according to Equation (38). From this result, firms have an incentive to impose excess labor on workers.

Because the optimal labor supply L_t^* is given by the worker's utility-maximization problem, we obtain the equilibrium price as follows:

$$p^* = w\left[\frac{L_t^*}{\left((1-\alpha)H_t\right)^{\left(\frac{1-\alpha}{\alpha}\right)}}\right]^{\left(\frac{1+\alpha}{1+2\alpha}\right)}. \tag{39}$$

The equilibrium quantity of the output Y_t^* is determined under the equilibrium price p^*.

Now, suppose that a firm thinks only of its profits and forces its workers to work excessively; that is, $L'_t > L_t^*$. Then, the profit in this period increases because of assumption $\pi'_{L_t} > 0$. Moreover, from the production function, production in this period Y'_t also increases.

From Proposition 7, workers increase their time-consuming behavior because of their increased labor supply; however, this effect reduces their sleep, which in turn reduces their health capital H_{t+1} in the next period. Therefore, from the production function, the output in the next period Y'_{t+1} is less than the equilibrium output Y^*_{t+1} even if workers supply the same amount of labor as L^*_{t+1}. However, as real wages decrease because of a decline in workers' health, firms' demand for labor increases, and firms force workers to work harder.

Thereby, supply increases more but this increase is because workers' real wages are decreased by two effects: decreasing hours slept and decreasing hours spent on time-consuming behavior. An increase in labor causes a decrease in real wages (Equation (37), subsequently reducing consumption and sleep. These results lead to excess supply, and firms cannot maximize their profits.

Moreover, we consider whether firms force workers to work excessively for only one period and then allow them to freely choose their hours worked from the next period onwards. Output in the first t period increases over the equilibrium output; however, output in the next period decreases because of lower workers' health capital. Even if the firm attempts to return production to equilibrium during period $t+2$, the necessary hours slept needed to recover workers' health capital are more than the initially increased hours spent on labor and time-consuming behavior because of the existence of the physical depreciation rate ϕ_t. Thus, the output in period $t+1$ decreases by more than the incremental amount of production in period t. Comparing the amount of output during these three periods, it holds that

$$Y_t^* + Y_{t+1}^* + Y_{t+2}^* > Y_t' + Y_{t+1}' + Y_{t+2}'. \tag{40}$$

Thus, firms cannot maximize their profits by forcing excessive work on workers.

From Equation (40), when L increases, p is an increasing function of L and a decreasing function of H. Thus, p increases by more than the incremental amount of L. In addition, because w is an increasing function of H_t and a decreasing function of L_t, increasing L decreases wages by more than the incremental amount of L. These results show that the real wage drastically decreases.

From Equation (29), $L_t > F_t$ holds in this state, and the only way to consume Y_t' is to borrow money.[16]

If workers go into debt, they must put in more labor to repay this debt because of the existence of interest rate r. However, if they work harder, their health continues to decline, and their wages continues to decrease. In this way, even if workers are able to pay off their debts, they still need more sleep to restore their health after this period. Then, the worker's total utility is reduced because they cannot supply enough labor; therefore, they cannot consume enough in the future.

Thus, when the firm forces workers to work in excess, the economy is in a state of excess supply, and workers are unable to maximize utility. Therefore, an externality exists in which an increase in work and time-consuming behavior time results in a decrease in sleep.

Hence, these results indicate that social welfare is not maximized when firms force workers to work excessively.

In this case, suppose the government enforces a policy of fixing working hours to $\bar{L}$.

[16] Therefore, increasing labor to increase time-consuming behavior results in excess output, Y', which cannot be consumed within the time constraint.

Then, at least the state in Section 3.3 can be achieved. Here, workers' health is improved because of an increase in sleep. The production function increases the firm's output. In addition, real wages increase because of improved health. An increase in real wages leads to an increase in consumption, which in turn increases workers' utility. Furthermore, an increase in consumption has a positive effect on health, and workers' health is further improved. An increase in consumption also implies an increase in firm profits. Of course, in the situation described in Section 3.3, the problem of excess supply still remains. However, if the increase in real wages due to health improvements is sufficiently high, the excess supply is infinitesimal.[17]

Therefore, policies that reduce labor not only improve workers' health but also improve social welfare.

4. Results

This section summarizes three results obtained from the previous sections.

First, we reviewed the relationship between age and sleep duration. Some empirical studies found that both men and women sleep less as they age. Propositions 1, 3, and 6 of our study generated results consistent with this statistical evidence. Our propositions can explain the phenomenon of diminished sleep with increasing age. Workers attempt to maintain their health throughout their lives by sleeping more than the optimal value at a young age because workers consider the presence of physical attrition. In addition, from Equation (37), workers intend to earn higher wages over their lifetimes and consume more in the future by storing their health when they are young.

Also interesting to note is that similar results were obtained for cases with and without budget constraints. These two cases can be interpreted as differences in initial assets. Even if the initial assets are sufficient, workers get more sleep when they are young because they obtain utility from their health. In other words, they choose to continue consuming under good health throughout their lives rather than consuming more when they are young and gaining utility from it.

Finally, Sections 3.2, 3.5, and 3.6 suggest conventions and statutory considerations that affect hours spent on sleep and time-consuming behavior by reducing the hours worked. If sleep supports health more than time-consuming behavior, workers can improve their health capital by sleeping. From Propositions 2 and 5, when the labor supply is constant, workers sleep more than when their work is flexible. From Proposition 7, we know that if workers attempt to increase consumption, then labor also increases, leading to a decrease in hours slept. From these results, workers' health is found to improve by maintaining constant hours worked. Furthermore, from the discussion in Section 3.6, if firms force workers to excessively supply labor, workers' health will decline from the damage to the body caused by labor; furthermore, the output will decrease. When workers' health declines, from Equation (39), prices increase as wages decrease, leading to an oversupply of goods because they cannot consume all goods within the time constraint. Then, the third

[17] If wages increase until $p < w$, then the situation will be the same as in Section 3.2.

inequality in Equation (29) is established, and workers sacrifice more sleep to consume goods. Moreover, they cannot consume goods enough within their time constraint if their real wages decrease.

In this paper, we assumed that time-consuming behavior improves workers' health; therefore, decreasing the hours slept and spent on time-consuming behavior leads to a decrease in their health and utility. If workers attempt to maintain sleep while increasing consumption by borrowing money, they will have to increase their labor in the next period to pay it off. Thus, they will be in a similar situation of sacrificing sleep to consume goods. Moreover, once they have borrowed money, workers will have to either increase their labor or reduce their consumption to repay the debt. In either case, their welfare is not maximized. If the government implements the policies that reduce hours worked, then, from Proposition 7, workers increase their hours slept. Moreover, the labor reduction policy then increases not only workers' health but also their consumption within their time constraint because the relation between consumption and labor approaches the second equation in Equation (29). Workers can consume enough goods through their increased wages as their health improves. Firms can also increase their profits because of their workers' improved health. Thus, when labor is in excess supply, policies that reduce working hours improve social welfare and not only workers' health.

5. Conclusions

The main purpose of this study was to construct a new health investment model that includes sleep and analyzes how sleep affects workers' health.

Grossman (1972) developed the first health investment model. The Grossman model analyzed health investment by health-related goods treated in the market, predominantly focusing on health services. However, sleep was not included in health-related goods in the model. Sleep does not fit Grossman's model because it is not traded in the market. In addition, sleep requires only time and cannot be valued in monetary terms, which makes a theoretical analysis difficult. In our study, we solved this problem by introducing sleep into the time constraint. Moreover, by assuming that consumption requires both time and money, we were able to include consumption in the time constraint, allowing us to analyze the balance between consumption, sleep, and labor.

The two main results of the model are as follows. The first result is that workers slept more hours when they were young and decreased their hours as they aged. This result is consistent with Ohayon et al. (2004) and Kuroda (2008), and the NIH's recommendations for sleep duration (NIH, 2006). However, these studies did not explain why the sleep duration followed this trend. Our model provides a theoretical explanation. We found that workers attempt to stock up on health capital by sleeping more at a younger age to maintain health throughout their lives. Workers conserve consumption and stockpile health capital at a younger age. As they age, they attempt to maximize utility by decreasing their hours slept and increasing their consumption. We also find that labor productivity and worker income are increasing functions of health capital. Furthermore, we find that price is a decreasing function of health capital. These results could also explain the sleep transition. That is, workers increase their income and production by storing health at a younger age, thereby allowing workers to consume more within the time constraint. The second result is that labor and consumption are complementary. In other words, if workers increase consumption, labor also simultaneously increases. If they decrease labor, consumption also decreases. This result shows that if a firm imposes excessive work on its workers, policies that reduce or keep hours worked constant can improve workers' health. When hours worked are reduced, workers are forced to consume less but sleep more; therefore, their health improves. In fact, by lengthening the DAILY REST PERIOD, workers may be able to sleep more and improve their health (Ikeda, et al., 2022). However, as health improves, production and workers' real wages increase. Then, workers can increase consumption, which in turn improves their utility. Moreover, consumption also improves workers' health; thus, health is improved through the effects of both increased sleep and consumption through a labor reduction policy. Additionally, increasing consumption leads to improving firms' profits. Thus, we find that policies that reduce hours worked can improve not only workers' health but also social welfare. This result is important in that it shows that health and sleep are essential to social welfare.

To be noted is that this study includes several simplified assumptions, all of which should be relaxed in future studies. First, this study does not consider goods that do not require time to consume. However, we cannot ignore the existence of health investments that can be made with little or no time, such as supplements. Introducing such goods into the model may be expected to allow us to analyze their balance with sleep. In addition, by including savings in our model, the balance between sleep, consumption, and labor may be analyzed more precisely.

Another limitation is the possibility of introducing uncertainty. Even if healthy, death can be caused by a sudden heart attack. Thus, we must construct a model in which the probability of death is introduced. The positive correlation between sleep deprivation and mortality is widely known; however, some statistics show that excessive sleep duration may actually increase mortality (Youngstedt and Kripke, 2004; Steptoe et al., 2006; Grandner et al., 2010b). However, as previously discussed, sleep also improves health. Therefore, a new model that identifies optimal sleeping time, in which sleep contributes to both recovering health and reducing mortality risk, should be considered.

In addition, this study only focused on workers' long-term decision making for sleep. However, in the short term, workers may temporarily sacrifice their sleep to work or increase their consumption. To analyze such behaviors, a model for analyzing the short-term sleep duration should also be considered.

Finally, this model can be extended to a life cycle model. In this study, workers are assumed to work until their final term and then die, which is unrealistic. Essentially, one of the incentives for workers to stay healthy is their desire to stay healthy during retirement. By making modifications with respect to these points, this model allows for a more general analysis.

However, irrespective of the modifications made, neglecting the essential features of the model presented in this study would be erroneous. The most important aspect of this study is that we have provided a constraint on sleep---a factor that is theoretically difficult to address---allowing for analysis in the theoretical model. Considering how to deal with goods that cannot be handled through the market rather than ignoring them is important.

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Appendix

This appendix considers the case that time-consuming behavior has a higher health-promotional effect than sleep and provides proof of our proposition and of selected theorems and lemmas.

When Time-Consuming Behavior Has Highly Health-Promoting Effects

In Section 3, we assumed that sleep has a higher effect on promoting health than time-consuming behavior. However, consumption may occur with a health-promoting effect that is higher than that of sleep, as described by $\beta > \alpha$.

If $\beta > \alpha$ holds, then in Sections 3.2 and 3.4, sleep approaches the optimal value from a lesser point than the optimal value. However, in such a case, time-consuming behavior approaches the optimum value from a point higher than the optimal point, which is the exact opposite of the results of the model described in Section 3. Indeed, sacrificing sleep and increasing time-consuming behavior in the era of young age seems natural if no budget constraints exist. We may also be able to reduce sleep through consumption, which is more health-improving than is sleep. For example, entering a hot spring, receiving a massage, or resting in an oxygen capsule may be more beneficial for health than sleeping.

If an individual is not restricted by budget constraints, they take the minimum sleep for living but can further reduce sleep to increase time-consuming behavior, which has a better health-promoting effect than sleep. However, when a budget constraint exists, we can confirm that even if $\beta > \alpha$ is satisfied but cannot necessarily obtain such a result. In the case in which labor supply is constant, as in Section 3.3, increasing consumption beyond the budget is not possible. If such a point could be chosen, it would only be when the initial assets were large enough, wage rates were sufficiently large, or prices were low enough. This is no different from the case without the budget constraints.

In Section 3.5, the following equations were obtained:

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{du}{dS} + \frac{dv}{dH} \cdot \left[\alpha - \left(\frac{1}{\left(1 + \frac{p}{w}\right)}\right)\left(\beta - \gamma\left(\frac{p}{w}\right)\right)\right]\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right] = 0,$$

$$\frac{\partial U}{\partial F_t} = \delta^{t-1} \cdot \frac{du}{dF} + \frac{dv}{dH} \cdot \left[\beta - \left(\alpha\left(1 + \frac{p}{w}\right) + \gamma\left(\frac{p}{w}\right)\right)\right]\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right] = 0,$$

even if $\beta > \alpha$, then we cannot obtain the result that an individual increases time-consuming behavior by reducing sleep. If labor is flexible and a budget constraint exists, we obtain the same results as in Section 3.5, even if $\beta > \alpha$. To obtain the result that workers increase their consumption by reducing their sleep, the following condition must be satisfied:

$$\alpha\left(1 + \frac{p}{w}\right) + \gamma\left(\frac{p}{w}\right) < \beta. \tag{41}$$

If this condition holds, we obtain Proposition 8.

Proposition 8. *Suppose that labor supply is flexible and a budget constraint exists. If $\beta > \alpha$, then the following inequality holds:*

$$\alpha\left(1+\frac{p}{w}\right)+\gamma\left(\frac{p}{w}\right)<\beta,$$

Then, the following inequalities hold.

$$S_1^* < S_2^* < \cdots < S_{T-1}^* < S_T^* = S^*;$$

$$F_1^* > F_2^* > \cdots > F_{T-1}^* > F_T^* = F^*;$$

$$L_1^* > L_2^* > \cdots > L_{T-1}^* > L_T^* = L^*.$$

Proof. See the Appendix.

However, inequality (41) is a relatively strict condition because if such goods exist, we can consider that the price of these goods would be $p \geq w$ relative to the wage rate of a typical worker. As Proposition 7 shows, increasing time-consuming behavior means more labor supply, and if the effect of restoring health from consumption is not greater than the damage to health from work, sleep cannot be reduced. Generally, goods that significantly restore health are expensive, and increasing the labor required to purchase them may result in poor health. Therefore, we need not consider the case in which prices are extremely low. Thus, we consider the case in which $\beta > \alpha$ and wage rates are sufficiently large. We can then rewrite these two equations as follows:

$$\frac{\partial U}{\partial S_t} = \delta^{t-1}\cdot\frac{du}{dS} - \frac{dv}{dH}\cdot[\beta-\alpha]\left[\delta^t + \sum_{j=t+1}^{T-1}\delta^j\prod_{k=t+1}^{j}\phi_k\right] = 0,$$

$$\frac{\partial U}{\partial F_t} = \delta^{t-1}\cdot\frac{du}{dF} + \frac{dv}{dH}\cdot[\beta-\alpha]\left[\delta^t + \sum_{j=t+1}^{T-1}\delta^j\prod_{k=t+1}^{j}\phi_k\right] = 0,$$

In this case, Proposition 8 is obtained.

Proposition 8 is consistent with the results for sleep and time-consuming behavior in Sections 3.2 and 3.4 under the assumption that $\beta > \alpha$, but its interpretation differs. If budget constraints and labor are flexible, increasing consumption requires more labor. However, disutility from labor (γ) no longer applies if the wage rate is sufficiently large.

This situation may arise among people who work as a pastime. They seek to recover their health not just to increase hours spent on time-consuming behavior but also to increase hours worked because, as Proposition 7 shows, labor supply and time-consuming behavior are complementary. In summary, if β is large, then more efficient ways exist of restoring health than sleeping. Those who like to work can derive more benefits. If $\alpha > \beta$ holds, then they sleep to recover health because they must work more to secure more consumption. Thus, they reduce hours slept by refreshing hours spent on time-consuming behavior, which increases the number of hours that they can work. However, as Proposition 8 shows, the optimal time for time-consuming behavior and labor in each period gradually approaches the optimal value from a point higher than the optimal value, even in individuals who like to work. We can interpret this as an age-related deterioration of the body ϕ_t. Therefore, such individuals cannot recover completely unless their sleep gradually increases. Moreover, if individuals can offset their labor-induced physical damage with effective consumption within a short period, then the health decrease with an increase in age can be ignored. However, the existence of such health-promoting time-consuming behavior and goods within a short period is not realistic.

The discussion thus far has shown that not only $\alpha > \beta$ but also the assumption of $\beta > \alpha$ can be considered. Apart from individuals who obtain disutility from labor supply, we should not forget that individuals who like to work also exist.

Proof of Lemma 1

Lemma 1. *One variable functionu* $(x, \bar{T} - x)$ *is concave.*

Proof. We put $G(x) = u(x, \bar{T} - x)$. We will show that the following equation holds for all variables $x, y \in R_{++}$ and $\lambda \in [0,1]$:

$$G(\lambda x + (1-\lambda)y) \geq \lambda G(x) + (1-\lambda)G(y).$$

Since by concavity of u, we have

$$\begin{aligned} G(\lambda x + (1-\lambda)y) &= u\big(\lambda x + (1-\lambda)y, \bar{T} - (\lambda x + (1-\lambda)y)\big) \\ &= u(\lambda x + (1-\lambda)y, ((\lambda + (1-\lambda)) - (\lambda(\bar{}\{T\} - x) \ + (1-\lambda)(\bar{}\{T\} - y)\ )) \\ &= u\big(\lambda(x, \bar{T} - x) + (1-\lambda)(y, \bar{T} - y)\big) \\ &\geq \lambda u(x, \bar{T} - x) + (1-\lambda)u(y, \bar{T} - y) = \lambda G(x) + (1-\lambda)G(y). \end{aligned}$$

Thus, we obtain

$$G(\lambda x + (1-\lambda)y) \geq \lambda G(x) + (1-\lambda)G(y).$$

Proof of Proposition 1

In this model, parameters δ^t, ϕ_t, α, and β are positive and the inequality $\alpha > \beta$ holds. In addition $\delta \cdot [\alpha - \beta]$ must be positive.

From Assumption (iii), $v(H_t) > 0$, the derivative of the utility function at the health capital is positive, and via Equation (8), we have this inequality:

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{du}{dS} = -\frac{dv}{dH} \cdot (\alpha - \beta) \left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k \right] < 0.$$

for all periods $t = 1,2, \dots, T-1$. From Assumption (ii), the following relation holds for hours slept during final period T and any prior period t:

$$S_t^* > S_T^* = S^*,$$

Finally, concavity means that the slope of this curve always decreases. Thus, the following inequality holds for all periods t:

$$S_t^* > S_{t+1}^*.$$

Hence, we conclude that

$$S_1^* > S_2^* > \cdots > S_{T-1}^* > S_T^* = S^*.$$

Proof of Theorem 1

Theorem 1. *If the following equations hold during all periods before \$T\$, that is,*

$$S_1^* = \overline{T'} - \frac{w\bar{L}}{p}$$

$$S_2^* = \overline{T'} - \frac{(1+r)A_1}{p} - \frac{w\bar{L}}{p}$$

$$\vdots$$

$$S_T^* = \overline{T'} - \frac{(1+r)A_{T-1} - A_T}{p} - \frac{w\bar{L}}{p},$$

then the optimal values of sleep and time-consuming behavior are consistent with having no budget constraint even if one exists.

Proof. On such a point, a person can choose optimal time-consuming behavior F_t^* during each period by choosing S_t^*. Thus, the theorem is proven.

Proof of Proposition 2

Proof. From inequality (18), we have

$$\overline{T'} - \frac{(1+r)A_{t-1} - A_t + w\bar{L}}{p} > \overline{T'} - \frac{(1+r)A_t - A_{t+1} + w\bar{L}}{p}.$$

From the definition of S_t^{b*}, we have

$$\overline{T'} - \frac{(1+r)A_{t-1} - A_t + w\bar{L}}{p} = S_t^{b*} > S_{t+1}^{b*} = \overline{T'} - \frac{(1+r)A_t - A_{t+1} + w\bar{L}}{p}.$$

The proposition is proven.

Proof of Proposition 3

Suppose $S_t^* > S_t^{b*}$ holds. From Assumption (ii), workers can improve utility by sleeping less and enjoying more time-consuming behavior. Thus, we obtain

$$S_t^* \le S_t^{b*},$$

for all periods $t = 1,2,\dots,T$.

Proof of Theorem 2

Theorem 2. *The optimal values in the cases with and without budget constraints do not* *coincide if the following inequalities hold during all periods:*

$$S_1^* < \bar{T}' - \frac{w\bar{L}}{p}$$

$$S_2^* < \bar{T}' - \frac{(1+r)A_1}{p} - \frac{w\bar{L}}{p}$$

$$\vdots$$

$$S_T^* < \bar{T}' - \frac{(1+r)A_{T-1} - A_T}{p} - \frac{w\bar{L}}{p}.$$

Proof. From Propositions 2 and 3, the following inequalities hold:

$$S_t^* \leq S_t^{b*} < S_{t-1}^{b*},$$

for all periods $t = 1,2, \dots, T$. If $S_t^* = S_t^{b*}$ holds, then Theorem 1 holds at this point. Thus, it can be ignored. From the monotonicity of \$u\$, the budget constraint is established as an equality. Then we have

$$\bar{T}' - \frac{(1+r)A_{t-1} - A_t}{p} - \frac{w\bar{L}}{p} = S_t^{b*}.$$

Therefore, we obtain

$$S_t^* < S_t^{b*} = \bar{T}' - \frac{(1+r)A_{t-1} - A_t}{p} - \frac{w\bar{L}}{p},$$

for all periods $t = 1,2, \dots, T$.

Proof of Proposition 4

Suppose that hours spent on sleep and time-consuming behavior are zero at period t. This means

$$\bar{T} = L_t.$$

Accumulated health at period t is

$$H_{t+1} = \phi_t H_t - \gamma L_t.$$

Thus, health is decreased as labor supply increases.

Moreover, utility is represented as the sum of $u(S_t, F_t)$ and $v(H_t)$. By working fewer hours, people can take more time-consuming behavior and/or sleep. Their utility is improved by working less.

Proof of Theorem 3

Theorem 3. *If the utility function satisfies monotonicity, then the budget constraint can be written for all* $t = 1,2,\dots,T$ *as:*

$$\bar{T} = S_t + \left(1 + \frac{p}{w}\right) F_t.$$

Proof. In general, the assets at period t are

$$A_t = w\left[\bar{T}\sum_{j=1}^{t} {}_tC_j\, r^{j-1} - \sum_{j=1}^{t}(1+r)^{j-1}\left(S_j + \left(1+\frac{p}{w}\right)F_j\right)\right].$$

The above equation can be rewritten as

$$\bar{T}\sum_{j=1}^{t} {}_tC_j\, r^{j-1} = \sum_{j=1}^{t}(1+r)^{j-1}\left(S_j + \left(1+\frac{p}{w}\right)F_j\right) + A_t.$$

From the monotonicity of the utility function, they obtain the maximum utility by $A_t = 0$, then the above equation is rewritten as:

$$\bar{T}\sum_{j=1}^{t} {}_tC_j\, r^{j-1} = \sum_{j=1}^{t}(1+r)^{j-1}\left(S_j + \left(1+\frac{p}{w}\right)F_j\right).$$

This is none other than that the following formulas are satisfied in each period:

$$\bar{T} = S_t + \left(1 + \frac{p}{w}\right) F_t.$$

In addition, we obtain the following corollary from Theorem 3.

Corollary 1. *Sleep, time-consuming behavior, and labor satisfy below for all periods* $t = 1,2,\dots,T$*:*

$$S_t > 0, F_t > 0, L_t > 0.$$

Corollary 2. *There is the following relation between time-consuming behavior and labor:*

$$L_t = \frac{p}{w} F_t.$$

Proof of Proposition 5

Proof. This is the same as the proof for Proposition 1.

Proof of Proposition 6

Proof. Workers can increase consumption by supplying labor more and sleeping less if labor is flexible. If $w \geq p$ holds, workers eventually can attain optimal time-consuming behavior in Section 3.2, F^*, then this result coincides with Proposition 3. Besides, if $w <$

p holds, then the optimal hours slept at period t in this case is equal to S_t^{b*}. Additionally, from Proposition 2 and 5, this result holds at optimal hours slept for each period. Thus, the proposition is proven.

Proof of Proposition 7

From Theorem 3 and Corollary 2, if labor supply increases, then time-consuming behavior also increases. Thus, this proposition is proven.

Proof of Proposition 8

Proof. Since Inequality (41) holds, then we have

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{du}{dS} = -\frac{dv}{dH} \cdot \left[\alpha - \left(\frac{1}{\left(1+\frac{p}{w}\right)}\right)\left(\beta - \gamma\left(\frac{p}{w}\right)\right)\right]\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right],$$

$$\frac{\partial U}{\partial F_t} = \delta^{t-1} \cdot \frac{du}{dF} = -\frac{dv}{dH} \cdot \left[\beta - \left(\alpha\left(1+\frac{p}{w}\right) + \gamma\left(\frac{p}{w}\right)\right)\right]\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right],$$

From the above, we then obtain the following inequalities:

$$S_1^* < S_2^* < \cdots < S_{T-1}^* < S_T^* = S^*;$$

$$F_1^* > F_2^* > \cdots > F_{T-1}^* > F_T^* = F^*.$$

In addition, from Theorem 3 and Corollary 2, we have

$$\frac{w}{p}L_1^* > \frac{w}{p}L_2^* > \cdots > \frac{w}{p}L_{T-1}^* > \frac{w}{p}L_T^* = \frac{w}{p}L^*.$$

Dividing this by $\frac{w}{p}$, we obtain

$$L_1^* > L_2^* > \cdots > L_{T-1}^* > L_T^* = L^*.$$

Thus, Proposition 8 is proven.