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Debt to GDP Ratio and inflation from the perspective of Functional Finance Theory and MMT

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Abstract

This paper will examine the relationship between budget deficit, inflation rate and debt to GDP ratio from the perspective of Functional Finance Theory and MMT (Modern Monetary Theory). Using a basic macroeconomic model in which the interest rate of government bonds is endogenously determined, with overlapping generations model in mind, mainly, we show the following results.

1. If the proportion of the savings consumed is smaller than one, the larger the budget deficit is, the larger the inflation rate is.

2. If the proportion of the savings consumed is smaller than one, the larger the inflation rate is, the smaller the debt to GDP ratio is. Therefore, excessive budget deficits cause inflation, which results in a smaller debt to GDP ratio. A large budget deficit is not associated with a high debt to GDP ratio.

3. The larger the budget deficit is, the weaker the condition on the proportion of savings that is consumed in order for the debt to GDP ratio not to diverge is.

Keywords: Debt to GDP ratio, Inflation, Functional Finance Theory, MMT

1. Introduction

One of the most commonly used conditions for examining fiscal stability is the Domar condition (Domar(1944), Yoshino and Miyamoto(2020)). The Domar condition compares the interest rate with the economic growth rate under balanced budget (excluding interest payments on the government bonds), and if the former is greater than the latter, public finance will become unstable, and the government debt to GDP ratio will continue to grow. Yoshino and Miyamoto(2020) try to modify the Domar condition by focusing not only on the supply side of government bonds but also on the demand side, while keeping the idea of

fiscal instability indicated by the Domar condition. However, our interest is different from that.

Using a simple macroeconomic model with peoples' money holding, we will present some results about debt to GDP ratio in a growing economy including the Domar condition from the perspective of Functional Finance Theory (Lerner(1943), Lerner(1944)) and MMT (Modern Money Theory or Modern Monetary Theory, Kelton(2020), Mitchell, Wray, Watts(2019), Wray(2015)[1]).

In the next section, we will show the following results.

1. The budget deficit including interest payments on the government bonds equals an increase in the savings from a period to the next period. (Proposition 1)

2. If the savings in the first period (Period 0) is positive (unless the savings are made solely through stocks), we need budget deficit to maintain full employment in a growing economy under constant prices or inflation in the later periods. (Proposition 2)

3. Excessive budget deficit induces inflation under full employment. (Proposition 3)

4. If the proportion of the savings consumed is smaller than one, the larger the budget deficit is, the larger the inflation rate is. (Proposition 4)

5. If the proportion of the savings consumed is smaller than one, the larger the inflation rate is, the smaller the debt to GDP ratio is. Therefore, excessive budget deficits cause inflation, which results in a smaller debt to GDP ratio. A large budget deficit is not associated with a high debt to GDP ratio. (Proposition 5)

6. The larger the budget deficit is, the smaller the possibility of the debt to GDP ratio diverging to infinity is. (Proposition 6)

In Section 3 we consider endogenous determination of the interest rate on the government bonds by the monetary policy of the government.

In Section 4 we examine the so-called Domar condition, and show the following results.

1. If the proportion of the savings consumed is larger than $\frac{1}{2}$, the debt to GDP ratio does not diverge infinity under balanced budget excluding interest payments. (Proposition 7)

2. The larger the budget deficit is, the weaker the condition on the proportion of savings that is consumed in order for the debt to GDP ratio not to diverge is. (Proposition 8)

This paper is one of the attempts to give a theoretical basis to Functional Finance Theory and MMT. In particular, we provide a rationale for the following claims (Kelton(2020)).

We refer to the summary of Kelton's book by Hogan(2021). In fact, Hogan argues that Kelton is wrong, but he summarizes Kelton's argument to the point.

[1] Japanese references of MMT are Mochizuki (2020), Morinaga (2020), Nakano (2020), Park (2020), Shimakura (2019).

1. "The treasury creates new money."

The money supply (or supply of government bonds) equals the savings. An increase in the money supply equals the increase in the savings. An increase in the savings from a period to the next period equals the budget deficit in the latter period. The rate of an increase in the savings, which equals the rate of in increase in the money supply, equals the rate of economic growth, and therefore the budget deficit and an increase in the money supply do not cause inflation in this case.

2. "Inflation is caused by federal government deficit spending, not by Fed policy."

If the actual budget deficit is larger than the budget deficit that is necessary and sufficient to maintain full-employment in an growing economy under constant price, the price of the good will rise.

3. "Federal government spending is not related to taxes or borrowing."

Sustained budget deficits are necessary to maintain full-employment in a growing economy, and these budget deficits make it possible to maintain full-employment. It is impossible to maintain full-employment in a growing economy with balanced budget. Therefore, if the budget deficit to maintain full-employment is financed by the national debt, it does not need to be repaid or redeemed, and must not be repaid or redeemed. Future budget surpluses need not and must not make up the deficits for growth.

2. Budget Deficit and Debt to GDP Ratio

Using a simple macroeconomic model we analyze budget deficit and the debt to GDP ratio. In a broad sense, savings are made by government bonds, money, and stocks, of which those made by government bonds and money are analyzed as savings in this paper. The amounts of government bonds and money supply are determined by the government. Although money does not earn interest and government bonds earn interest, consumers are willing to hold a certain amount of money for reasons such as the liquidity of money. The holding of money is considered to be a decreasing function of the interest rate of the government bonds (while the holding of government bonds is an increasing function of the interest rate). The reasons for this are as follows. This part implicitly assumes an overlapping generations model in which people live for two periods[2]. People decide how much money and government bonds to hold so that the marginal utility of holding one more unit of money and the marginal utility of interest income from holding government bonds are equalized. Since the marginal utility of money decreases as the amount of money held increases, the amount of money held is a decreasing function of the interest rate of the government bonds.

The share of government bonds in savings is denoted by $b(r)$, $0 < b(r) \leqq 1$. r is the interest rate of the government bonds. $b(r)$ is an increasing function of r. The share of money in savings is $1 - b(r)$. The investment is financed by savings in the form of stocks,

[2] There are some studies, for example, Tanaka (2021a), Tanaka (2021b) and Tanaka (2021c) which use an overlapping generations model, according to the model by M. Otaki such as Otaki (2007), Otaki (2009), Otaki (2015), to analyze the problem of budget deficit in a growing economy

and it may be a decreasing function of the interest rate of the government bonds. However, for simplicity we assume that the investment is constant in each period. The interest rate of the government bonds is endogenously determined by the monetary policy of the government.

2.1 Period 0

First consider Period 0 at which the world starts. All variables represent nominal values. Let Y_0, C_0, I_0, T_0 and G_0 be the GDP, consumption, investment, tax and fiscal spending in Period 0. Then,

$$Y_0 = C_0 + I_0 + G_0.$$

The consumption is written as

$$C_0 = \bar{C}_0 + \alpha(Y_0 - T_0), \qquad 0 < \alpha < 1.$$

α is the propensity to consume. $\bar{C}_0$ is the constant part of consumption in Period 0. It is financed by the savings carried over from the previous period. Since there is no previous period of Period 0,

$$\bar{C}_0 = 0.$$

Then,

$$C_0 = \alpha(Y_0 - T_0),$$

and

$$Y_0 = \alpha(Y_0 - T_0) + I_0 + G_0.$$

From this

$$(1 - \alpha)(Y_0 - T_0) = I_0 + G_0 - T_0.$$

The savings in Period 0, which is carried over to the next period, is

$$S_0 = (1 - \alpha)(Y_0 - T_0) - I_0.$$

Therefore, we have

$$G_0 - T_0 = (1 - \alpha)(Y_0 - T_0) - I_0 = S_0.$$

Let us assume full employment in Period 0, and denote the full employment GDP by Y_f, that is,

$$Y_0 = Y_f.$$

Then, we obtain

$$G_0 - T_0 = (1 - \alpha)(Y_f - T_0) - I_0 = S_0. \qquad (1)$$

This is the budget deficit we need to achieve full employment in Period 0. It is determined by Y_f, I_0 and T_0. From this we get the following equation.

$$G_0 = (1-\alpha)(Y_f - T_0) + T_0 - I_0.$$

This is the fiscal spending needed to achieve full employment given T_0 and I_0.

Unless the savings are made solely through stocks, (1) is positive.

If the budget deficit is larger than the value in (1), and tax and investment also increase, then Y_f increases and (1) still holds. The budget deficit (or the fiscal spending given tax) determines the nominal value of the full employment GDP. We will use it as the basis for subsequent GDP values.

2.2 Period 1

Next, consider Period 1. Again all variables represent nominal values. Let Y_1, C_1, I_1, T_1 and G_1 be the GDP, consumption, investment, tax and fiscal spending in Period 1. Then,

$$Y_1 = C_1 + I_1 + G_1.$$

The consumption is written as

$$C_1 = \bar{C_1} + \alpha(Y_1 - T_1).$$

$\bar{C_1}$ is the constant part of consumption in Period 1. It is financed by the savings carried over from Period 0. Let δ be the proportion of the savings consumed. Then,

$$\bar{C_1} = \delta(1 + b(r)r)S_0, \qquad 0 < \delta \leqq 1,$$

and

$$C_1 = \delta(1 + b(r)r)S_0 + \alpha(Y_1 - T_1).$$

So,

$$Y_1 = \delta(1 + b(r)r)S_0 + \alpha(Y_1 - T_1) + I_1 + G_1.$$

From this

$$(1-\alpha)(Y_1 - T_1) = \delta(1 + b(r)r)S_0 + I_1 + G_1 - T_1.$$

Therefore,

$$G_1 - T_1 = (1-\alpha)(Y_1 - T_1) - I_1 - \delta(1 + b(r)r)S_0.$$

The savings in Period 1, which is carried over to Period 2, is

$$S_1 = (1-\alpha)(Y_1 - T_1) - I_1 + (1-\delta)(1 + b(r)r)S_0.$$

This means

$$G_1 - T_1 = S_1 - (1 + b(r)r)S_0.$$

Alternatively,

$$G_1 - T_1 + b(r)rS_0 = S_1 - S_0. \tag{2}$$

Therefore, the budget deficit in Period 1 equals the increase in the savings from Period 0 to

Period 1.

2.3 Growth and Inflation in Period 1

We assume that the economy grows by technological progress, and assume full employment. The real growth rate is $g > 0$. Also the prices may rise from Period 0 to Period 1, that is, there may be inflation. Let p be the inflation rate. Then,

$$(1+g)(1+p)-1 = g+p+gp$$

is the nominal growth rate.

Under nominal growth at the rate of $g+p+gp$,

$$Y_1 = (1+g)(1+p)Y_f.$$

Tax and investment also increase at the same rate as follows under the assumption that inflation is predicted,

$$T_1 = (1+g)(1+p)T_0, \qquad I_1 = (1+g)(1+p)I_0.$$

Then, the savings in Period 1 is

$$S_1 = (1-\alpha)(1+g)(1+p)(Y_f - T_0) - (1+g)(1+p)I_0 + (1-\delta)(1+b(r)r)S_0.$$

It is rewritten as

$$S_1 = (1+g)(1+p)S_0 + (1-\delta)(1+b(r)r)S_0. \tag{3}$$

If $\delta < 1$, we have

$$S_1 > (1+g)(1+p)S_0. \tag{4}$$

From (2) and (3),

$$G_1 - T_1 + b(r)rS_0 = (1+g)(1+p)S_0 + [b(r)r - \delta(1+b(r)r)]S_0, \tag{5}$$

and

$$G_1 - T_1 = (1+g)(1+p)S_0 - \delta(1+b(r)r)S_0 < (1+g)(1+p)(G_0 - T_0). \tag{6}$$

They are the budget deficits, with or without interest payments on the government bonds, which we need to achieve full employment in Period 1 under nominal growth at the rate of $g+p+gp$. If

$$\begin{aligned} G_1 - T_1 + b(r)rS_0 &= (1+g)S_0 + [b(r)r - \delta(1+b(r)r)]S_0 \\ &< (1+g)(1+p)S_0 + [b(r)r - \delta(1+b(r)r)]S_0, \end{aligned}$$

the economy grows at the real growth rate g without inflation. Therefore, we can say that excessive budget deficit induces inflation.

From (3) the debt tp GDP ratio in Period 1 is

$$\frac{S_1}{Y_1} = \frac{(1+g)(1+p)+(1-\delta)(1+b(r)r)}{(1+g)(1+p)Y_f} S_0 \tag{7}$$

$$= \frac{(1+g)(1+p)+(1-\delta)(1+b(r)r)}{(1+g)(1+p)Y_f}[(1-\alpha)Y_f - I_0] = \frac{S_0}{Y_0} + \frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}\frac{S_0}{Y_0}.$$

If $\delta < 1$, the larger the value of p, the smaller $\frac{S_1}{Y_1}$ is. On the other hand, (5) means that the larger the budget deficit is, the larger the value of p, given S_0, r, g and δ, is .

2.4 Period 2

Next, consider Period 2. Also in this subsection all variables represent nominal values. Let Y_2, C_2, I_2, T_2 and G_2 be the GDP, consumption, investment, tax and fiscal spending in Period 2. Then,

$$Y_2 = C_2 + I_2 + G_2.$$

The consumption is

$$C_2 = \bar{C}_2 + \alpha(Y_2 - T_2).$$

$\bar{C}_2$ is the constant part of consumption in Period 2. It is financed by the savings carried over from Period 1. Similarly to the case of Period 1,

$$\bar{C}_2 = \delta(1+b(r)r)S_1,$$

and

$$C_2 = \delta(1+b(r)r)S_1 + \alpha(Y_2 - T_2).$$

So,

$$Y_2 = \delta(1+b(r)r)S_1 + \alpha(Y_2 - T_2) + I_2 + G_2.$$

From this

$$(1-\alpha)(Y_2 - T_2) = \delta(1+b(r)r)S_1 + I_2 + G_2 - T_2.$$

Therefore,

$$G_2 - T_2 = (1-\alpha)(Y_2 - T_2) - I_2 - \delta(1+b(r)r)S_1.$$

The savings in Period 2, which is carried over to Period 3, is

$$S_2 = (1-\alpha)(Y_2 - T_2) - I_2 + (1-\delta)(1+b(r)r)S_1.$$

This means

$$G_2 - T_2 = S_2 - (1+b(r)r)S_1.$$

Alternatively,

$$G_2 - T_2 + b(r)rS_1 = S_2 - S_1. \quad (8)$$

Therefore, the budget deficit in Period 2 equals the increase in the savings from Period 1 to Period 2.

2.5 Growth and Inflation in Period 2

Again we suppose that the economy nominally grows by technological progress and inflation at the rate of $g + p + gp$, then

$$Y_2 = (1+g)^2(1+p)^2 Y_f.$$

We assume that the inflation rate p is constant. Tax and investment also increase at the same rate as follows,

$$T_2 = (1+g)^2(1+p)^2 T_0, \qquad I_1 = (1+g)^2(1+p)^2 I_0.$$

Then, the savings in Period 2 is

$$S_2 = (1-\alpha)(1+g)^2(1+p)^2(Y_f - T_0) - (1+g)^2(1+p)^2 I_0 + (1-\delta)(1+b(r)r)S_1.$$

It is rewritten as

$$\begin{aligned} S_2 &= (1+g)(1+p)S_1 + (1-\delta)^2(1+b(r)r)^2 S_0 \\ &= (1+g)^2(1+p)^2 S_0 + (1-\delta)(1+b(r)r)S_1. \end{aligned} \tag{9}$$

If $\delta < 1$, we have

$$S_2 > (1+g)(1+p)S_1. \tag{10}$$

From (8) and (9) we obtain

$$G_2 - T_2 + b(r)rS_1 = (1+g)^2(1+p)^2 S_0 + [b(r)r - \delta(1+b(r)r)]S_1. \tag{11}$$

and

$$G_2 - T_2 = (1+g)^2(1+p)^2 S_0 - \delta(1+b(r)r)S_1. \tag{12}$$

Since

$$G_1 - T_1 = (1+g)(1+p)S_0 - \delta(1+b(r)r)S_0,$$

and

$$S_1 > (1+g)(1+p)S_0,$$

we get

$$G_2 - T_2 < (1+g)(1+p)(G_1 - T_1).$$

(11) and (12) are budget deficits, with or without interest payments on the government bonds, we need to achieve full employment in Period 2.

By (3) and (9),

$$\begin{aligned} S_2 = [&(1+g)^2(1+p)^2 + (1-\delta)(1+b(r)r)(1+g)(1+p) \\ &+(1-\delta)^2(1+b(r)r)^2]S_0. \end{aligned} \tag{13}$$

If

$$G_2 - T_2 = (1+g)^2 S_0 - \delta(1+b(r)r)S_1 < (1+g)^2(1+p)^2 S_0 - \delta(1+b(r)r)S_1,$$

the economy grows at the real growth rate g without inflation. Therefore, we can say that excessive budget deficit induces inflation.

From (9) the debt tp GDP ratio in Period 2 is

$$\frac{S_2}{Y_2} = \frac{(1+g)^2(1+p)^2 S_0 + (1-\delta)(1+b(r)r)S_1}{(1+g)^2(1+p)^2 Y_f} \tag{14}$$

$$= \frac{S_0}{Y_0} + \frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)} \frac{S_1}{Y_1}$$

If $\delta < 1$, the larger the value of p, the smaller $\frac{S_2}{Y_2}$ is. On the other hand, (11) means that the larger the budget deficit $(G_2 - T_2)$ is, the larger the value of p, given S_0 and S_1, r, g and δ, is.

2.6 Period 3 and Beyond with Growth and Inflation

Note that we assume that the inflation rate is constant. It may be zero. By similar reasoning for Period 3, we get

$$G_3 - T_3 = S_3 - (1+b(r)r)S_2.$$

and

$$G_3 - T_3 + b(r)rS_2 = S_3 - S_2. \tag{15}$$

The savings in Period 3 is

$$S_3 = (1+g)^3(1+p)^3 S_0 + (1-\delta)(1+b(r)r)S_2 \tag{16}$$

$$= (1+g)(1+p)S_2 + (1-\delta)^3(1+b(r)r)^3 S_0.$$

Thus,

$$G_3 - T_3 + b(r)rS_2 \tag{17}$$

$$= (1+g)^3(1+p)^3 S_0 + [b(r)r - \delta(1+b(r)r)]S_2,$$

and

$$G_3 - T_3 = (1+g)^3(1+p)^3 S_0 - \delta(1+b(r)r)S_2. \tag{18}$$

(17) and (18) are budget deficits, with or without interest payments on the government bonds, which we need to achieve full employment in Period 3.

From (13) and (16), we get

$$S_3 = [(1+g)^3(1+p)^3 + (1-\delta)(1+b(r)r)(1+g)^2(1+p)^2$$
$$+(1-\delta)^2(1+b(r)r)^2(1+g)(1+p)$$

$$+(1-\delta)^3(1+b(r)r)^3]S_0.$$

Proceeding with this argument, we obtain the following result for Period n, $n \geqq 1$.

$$G_n - T_n + b(r)rS_{n-1} = S_n - S_{n-1} \tag{19}$$
$$= (1+g)^n(1+p)^nS_0 + [b(r)r - \delta(1+b(r)r)]S_{n-1}.$$

With or without inflation in Period n, we have

$$S_n = [(1+g)^n(1+p)^n \tag{20}$$
$$+(1-\delta)(1+b(r)r)(1+g)^{n-1}(1+p)^{n-1} + \cdots$$
$$+(1-\delta)^{n-1}(1+b(r)r)^{n-1}(1+g)(1+p)$$
$$+(1-\delta)^n(1+b(r)r)^n]S_0.$$

Similarly, for Period $n-1$,

$$S_{n-1} = [(1+g)^{n-1}(1+p)^{n-1} \tag{21}$$
$$+(1-\delta)(1+b(r)r)(1+g)^{n-2}(1+p)^{n-2} + \cdots$$
$$+(1-\delta)^{n-2}(1+b(r)r)^{n-2}(1+g)(1+p)$$
$$+(1-\delta)^{n-1}(1+b(r)r)^{n-1}]S_0.$$

Then,

$$S_n = (1+g)^n(1+p)^nS_0 + (1-\delta)(1+b(r)r)S_{n-1}$$
$$= (1+g)(1+p)S_{n-1} + (1-\delta)^n(1+b(r)r)^nS_0.$$

Thus, if $\delta < 1$

$$S_n > (1+g)(1+p)S_{n-1}. \tag{22}$$

Since

$$Y_n = (1+g)(1+p)Y_{n-1},$$

(20) and (21) mean

$$\frac{S_n}{Y_n} = \frac{S_0}{Y_0} + \frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}\frac{S_{n-1}}{Y_{n-1}}. \tag{23}$$

If $\delta < 1$, the larger the value of p, the smaller $\frac{S_n}{Y_n}$ is. On the other hand, (19) means that the larger the budget deficit is, the larger the value of p, given S_0 and S_{n-1}, r and δ, is.

2.7 Some Propositions

From (2), (8), (15) and (19) we obtain the following proposition.

Proposition 1 The budget deficit including interest payments on the government bonds equals an increase in the savings from a period to the next period.

(2), (4), (8), (10), (19) and (22) mean the following result.

Proposition 2 *If the savings in the first period (Period 0)* S_0 *is positive (unless the savings are made solely through stocks), we need budget deficit to maintain full employment under constant prices or inflation in the later periods.*

About inflation we found

Proposition 3 *Excessive budget deficit induces inflation under full employment.*

By (5), (7), (11), (14), (17), (19) and (23), we get

Proposition 4 *If* $\delta < 1$, *that is, the proportion of the savings consumed is smaller than one, the larger the budget deficit is, the larger the inflation rate is.*

And

Proposition 5 *If* $\delta < 1$, *the larger the inflation rate is, the smaller the debt to GDP ratio is.*

Therefore, excessive budget deficits cause inflation, which results in a smaller debt to GDP ratio. A large budget deficit is not associated with a high inflation rate.

2.8 Convergence and Divergence of Debt to GDP Ratio

From (20), we obtain

$$\frac{S_n}{Y_n} = \left[1 + \frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)} + \cdots + \left(\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}\right)^{n-1} + \left(\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}\right)^{n}\right]\frac{S_0}{Y_0}.$$

If

$$\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)} < 1, \tag{24}$$

we get

$$\frac{S_n}{Y_n} \to \frac{1}{1 - \frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}}\frac{S_0}{Y_0}$$

Then, the debt to GDP ratio $\frac{S_n}{Y_n}$ converges to a finite value. On the other hand, if

$$\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)} > 1, \tag{25}$$

the debt to GDP ratio diverges to infinity. Since the larger the budget deficit is, the larger the inflation rate p is, (24) and (25) mean that the larger the budget deficit is, the smaller the possibility of the debt to GDP ratio diverging to infinity is. We have shown the following proposition.

Proposition 6

The larger the budget deficit is, the smaller the possibility of the debt to GDP ratio diverging to infinity is.

3. Determination of Interest Rate

The demand for money in Period 0 is

$$(1 - b(r))S_0.$$

Denote the money supply by M_0. Then, r is determined so that

$$(1 - b(r))S_0 = M_0$$

is satisfied. Similarly, let M_n be the money supply in Period n. Then, the interest rate in Period n is determined so that

$$(1 - b(r))S_n = M_n$$

is satisfied. As the money supply M_n increases, r and $b(r)$ must be smaller. Therefore, an increase in the money supply lowers the interest rate, and also interest payment $b(r)rS_n$ decreases. This is the effect of monetary policy.

4. About Domar Condition

From the above discussion, the interest rate can be changed by monetary policy so that the so-called Domar condition (Domar(1944), Yoshino and Miyamoto(2020)), that the interest rate must be less than the economic growth rate to prevent the ratio of government debt to GDP from becoming infinitely large (in particular, if a balanced budget is achieved excluding interest payments on government bonds), can be satisfied, but even if this condition is not satisfied, the ratio of government debt to GDP may not become infinitely large. When savings are made in both government bonds and money, the issue is not the interest rate on government bonds itself, but the product of the share of savings held in government bonds and the interest rate on government bonds $b(r)r$ and the proportion of the savings consumed δ. We call

$$\delta(1 + b(r)r) - 1,$$

the adjusted interest rate. Since $\delta \leqq 1$ and $b(r) \leqq 1$, It is not larger than r.

Let us assume balanced budget excluding interest payments on the government bonds in Period 1 as follows.

$$G_1 - T_1 = 0.$$

Then, (6) means that the following equation must hold.

$$(1 + g)(1 + p) = \delta(1 + b(r)r). \tag{26}$$

Then, the nominal growth rate $g + p + gp$ equals the adjusted interest rate. With (26), (24) is reduced to

$$\delta > \frac{1}{2}. \tag{27}$$

For the periods after Period 1 we obtain similar results. Then, we get the following proposition.

Proposition 7 If the proportion of the savings consumed is larger than $\frac{1}{2}$, the debt to GDP ratio does not diverge infinity under balanced budget excluding interest payments even if the adjusted interest rate is larger than the real growth rate, that is,

$$1 + g < \delta(1 + b(r)r).$$

If there is budget surplus excluding interest payments, that is, $G_1 - T_1 < 0$,

$$(1 + g)(1 + p) < \delta(1 + b(r)r).$$

In this case the condition for the debt to GDP ratio not to diverge is more stringent than (27).

On the other hand, if there is budget deficit excluding interest payments, that is, $G_1 - T_1 > 0$,

$$(1 + g)(1 + p) > \delta(1 + b(r)r).$$

In this case the condition for the debt to GDP ratio not to diverge is less stringent than (27).

Summarizing the results,

Proposition 8 The larger the budget deficit is, the weaker the condition on the proportion of savings that is consumed in order for the debt to GDP ratio not to diverge is.

5. Conclusion

Using a simple macroeconomic model with peoples' money holding we have examined the relationship between budget deficit, inflation rate and debt to GDP ratio from the perspective of Functional Finance Theory and MMT. The main results are as follows.

1. If the proportion of the savings consumed is smaller than one, the larger the budget deficit is, the larger the inflation rate is. (Proposition 4)

2. If the proportion of the savings consumed is smaller than one, the larger the inflation rate is, the smaller the debt to GDP ratio is. Therefore, excessive budget deficits cause inflation, which results in a smaller debt to GDP ratio. A large budget deficit is not associated with a high debt to GDP ratio. (Proposition 5)

3. The larger the budget deficit is, the weaker the condition on the proportion of savings that is consumed in order for the debt to GDP ratio not to diverge is. (Proposition 6)

In future studies, we intend to incorporate endogenous growth theory into our analysis, using a model in which investment and growth rates are endogenously determined. However, the basic conclusions will remain the same.

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